A New and Novel Approach for Understanding and Flying a Precision Turn Around a Point

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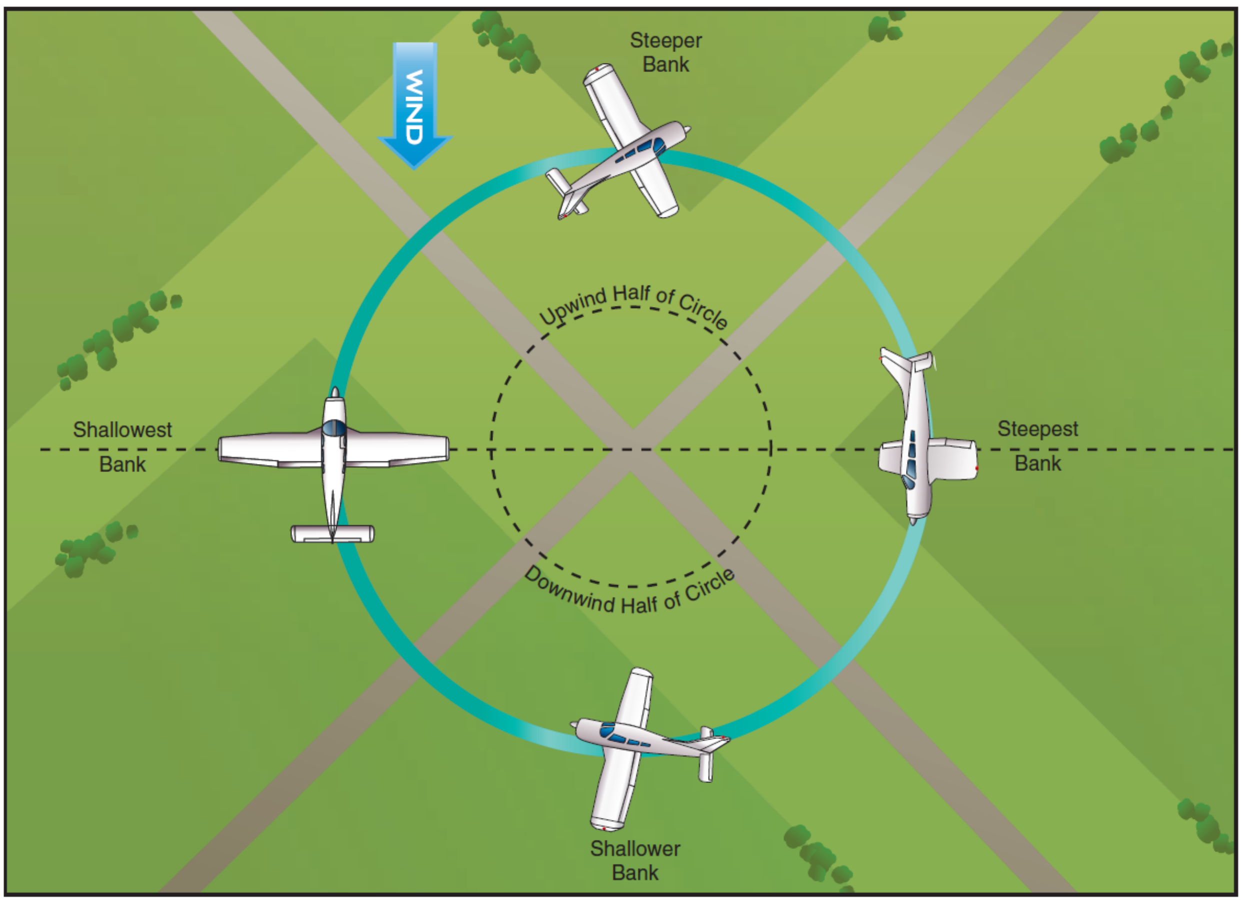

Summary

As a requirement in the certification for the Private Pilot Certificate, the Candidate must be proficient in the execution of three Ground Reference maneuvers. These are: (a) Rectangular Course, (b) S-Turns Across a Road, and (c) Turns Around a Point. All three maneuvers depend on understanding how wind affects the ground track of the aircraft.

In most texts on performing ground reference maneuvers, the subject of Tracking a Road, is treated fairly well. However, when it comes to the Turn Around a Point, the discussion is not very detailed. As an example, the FAA "Flight Training Handbook" indicates that the maximum bank angle will occur when the aircraft is on the downwind and the minimum bank angle will occur when the aircraft is on the headwind. In addition, the Handbook states, "Thus, if a maximum bank angle of 45 degrees is desired, the initial bank will be 45 degrees, if the aircraft is at the correct distance from the point. Thereafter, the bank is shallowed gradually until the point is reached where the aircraft is headed directly upwind. At this point the bank should be gradually steepened until the steepest bank is again attained when heading downwind at the initial point of entry." Thus, it is left to the Pilot as to what is mean by gradually changing the bank angle and what should be the radius of the turn that allows the maximum bank angle to be no more than 45 degrees on the downwind.

In order to answer these questions and provide the Flight Instructor and the Pilot with the very information he/she needs to fly the Turn Around a Point maneuver with precision, we derive the exact solution to the Turn Around a Point maneuver The exact solution provides the following information to the Pilot: (1) Rate of turn, (2) Bank angle, (3) Wind correction angle, and (4) Groundspeed ratio (i.e., the ratio of the groundspeed to the TAS), as a function of the angular position of the aircraft around the circle. We show that the solution is dependent on a single parameter, the wind speed ratio (i.e., V_{Wind}/V_{TAS}), with the rate of turn around the circle proportional to the square of the groundspeed ratio (i.e., V_G/V_{TAS}).

However, what is most intriguing is the solution also provides a crucial piece of information to the Pilot, which is the "gradient bank angle". The "gradient bank angle" is the number of degrees the bank angle must change per degree of angular displacement around the circle, as a function of where the aircraft is relative to the tailwind position. The "gradient bank angle" is found to be proportional to the wind speed ratio. The resultant behavior of the variation of the "gradient bank angle" is one of the root causes of why Pilots have trouble flying the maneuver with a wind of 10 knots or more.

If we follow the aircraft around the circle starting at the point when the aircraft is on the tailwind, the change in bank angle is zero, i.e. the bank is being held constant at its maximum value as we pass the downwind point. As we move toward the downwind side, the bank angle is being decreased; however, the rate at which the bank angle is decreased is increasing and reaches a maximum near the crosswind point (on the downwind side). For winds speed ratios of 0.3 or less, this maximum value of the “gradient bank angle” is less than 3 degrees per every 10 degrees of angular movement around the circle. During the time the aircraft is progressing from the crosswind point to the headwind point, the bank angle is continuing to decrease; however, the rate of change is also decreasing, until at the headwind point, the bank angle is being held constant at its minimum value. As the aircraft progresses from the headwind point to the opposite crosswind point (i.e. upwind side), the bank angle is increasing, with the rate increasing until it reaches a maximum in the neighborhood of the crosswind point. Again, this maximum in “gradient bank angle” is less than 3 degrees per every 10 degrees of angular movement around the circle. As the aircraft passes the crosswind point, the bank angle continues to increase with the rate of change decreasing until the bank angle is at maximum again on the tailwind, at which time, the bank angle is being held constant at its maximum value for a short period of time. This process than repeats itself.

Since the “gradient bank angle” is the key to the precise execution of the Turn Around a Point, we show that the “gradient groundspeed” parameter, which is the rate of change in the groundspeed around the circle, closely replicates the “gradient bank angle” parameter, and thus can be used as a guide in how to adjust the rate of change of the bank angle around the circle. In addition, we show that there are two primary variables the Pilot needs to observe around the circle. These are the radius of the turn and the groundspeed. The wind correction angle is shown not to be a primary variable since if the radius is correct, the wind correction angle must also be correct.

It is important to understand that these bank angle changes for wind speed ratios less than or equal to 0.3 are less than 3 degrees for every 10 degrees around the circle, and thus the term “gradually” is now quantified. In addition, the exact solution of the Turn Around a Point has been used to determine the radius of the turn which will ensure that the maximum bank angle is never greater than 45 degrees at the downwind point.

Using the information in this White Paper will allow both the Flight Instructor and Pilot not only to be able to fly a more precise Turn Around a Point, but have a better understanding of what is taking place during the execution of the maneuver.

1.0 Introduction

Ground reference maneuvers are maneuvers for both Private Pilot and Commercial Pilot Certificates. In the case of the Private Pilot Certificate, the pilot must satisfactorily demonstrate (a) Rectangular Course, (b) S-Turns Across a Road, and (c) Turns Around a Point. In the case of the Commercial Pilot certificate, the pilot must satisfactorily demonstrate Eights-On-Pylons. In this White Paper we concentrate on understanding and flying the Turn Around a Point. In a future White Paper, we will concentrate on understanding and flying the On-Pylon Turn.

As Flight Instructors, we attempt to break a maneuver down into basic elements which we can build on as we move from one maneuver to the next. If we look at the Rectangular Course, it is clear that this maneuver contains four straight legs and four 90 degree turns. In the case of S-Turns Across a Road, the maneuver is composed of two half circles of equal radius, in the first half the aircraft is turning in one direction and then in the second half it is turning in the opposite direction. Finally, in the case of a Turn Around a Point, the maneuver requires the pilot to maintain a constant radius around a specific point on the ground. It would appear that these maneuvers can be broken down into two basic elements, (a) Tracking a Road, and (b) Turns Around a Point. However, as we will see shortly, all the maneuvers for the Private Pilot Certificate are based on only one basic element and that is Tracking a Road. Thus, in order for the Student Pilot to comprehend and satisfactorily demonstrate the required ground reference maneuvers, the Student must understand wind drift and wind drift corrections, and thus the concept of the “Wind Triangle”. Understanding the “Wind Triangle Problem” allows the Student Pilot to understand the important parameters that affect the ground track of the aircraft in the presence of a wind.

In Section 2.0 we review the solution of the “Wind Triangle Problem”, which will provide us with the important parameters that affect the ground track in the presence of a wind. In Section 3.0 we discuss the simple concept of the “Dynamics of a Turn”, and the relationship between TAS, bank angle, and turn radius. In addition, we show how the rate of turn relates to bank angle and TAS. In Section 4 we discuss how wind affects the ground track during turning flight. In Section 5 we show how to relate the solution of the “Wind Triangle Problem” to the “Turn Around a Point”, which allows us to obtain the exact solution to the Turn Around a Point maneuver. In Section 6 we discuss how to precisely fly the “Turn Around A Point” maneuver and the primary visual cues that the Pilot should be focused on during the maneuver. Finally, in Section 7 we discuss how to select the proper radius of the turn to ensure that the bank angle on the tailwind never exceeds 45 degrees, as well as procedures for allowing the Pilot to enter the maneuver at points other than on the tailwind. In Section 8 we summarize the main points of this White Paper.

2.0 The Exact Solution of the "Wind Triangle Problem"

When a Pilot is attempting to fly over a straight road while correcting for wind drift, he/she is seeking a wind correction angle which will allow the aircraft to track directly over the road. Figure 1, taken from the FAA-H-8083-3A ("Airplane Flying Handbook"), shows the effect of wind drift on the aircraft attempting to track over a road. As can be seen, in order to track directly over the road the aircraft must establish a wind correction angle (WCA) into the wind.

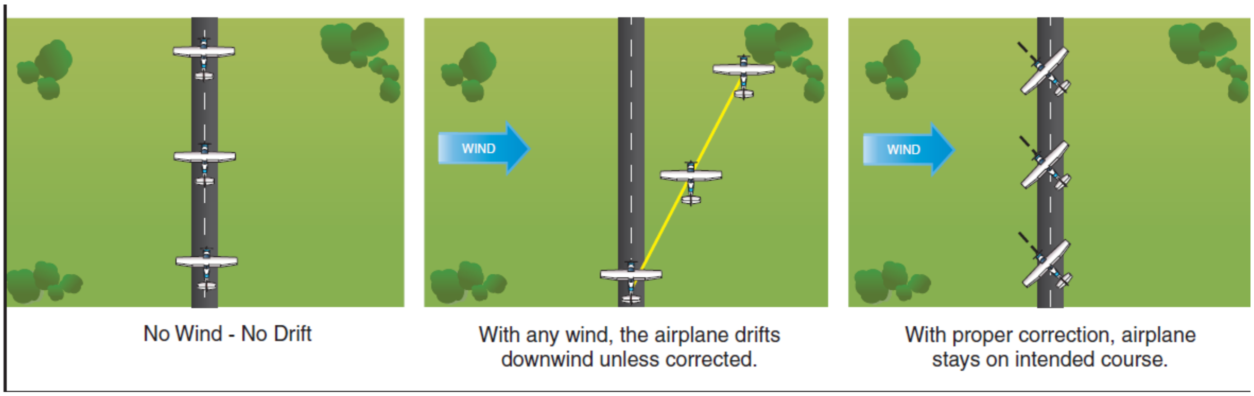

Figure 1 – Effects of Wind Drift on Tracking a Road

As a Student Pilot, we have been instructed in the use of either an E6B Wheel or an Electronic E6B Flight Computer to determine the WCA and the groundspeed along the course we are trying to track. The solution of the "Wind Triangle Problem" requires the Pilot to input the following four quantities: (a) Course to track, (b) True airspeed, (c) Wind direction, and (d) Wind speed. The course and wind direction can be input in either True or Magnetic, but both wind direction and course must be use the same reference. The solution of the "Wind Triangle Problem" provides the WCA and the groundspeed along the course to be tracked. The problem with the E6B is that it does not provide any information on what the important parameters are that control the WCA and the groundspeed. In order to obtain this information it is necessary to determine the exact solution of the "Wind Triangle Problem".

Figure 2 shows the basic "Wind Triangle" concept. Given the wind speed, V_{Wind}, the angle between the wind direction and the course line to be tracked, α and the true airspeed, V_{TAS}, we need to determine the WCA, σ and the groundspeed, V_G. Note that we do not require the individual values of the course to be tracked and the wind direction, but only the relative angle between the course and the wind direction, i.e., α.

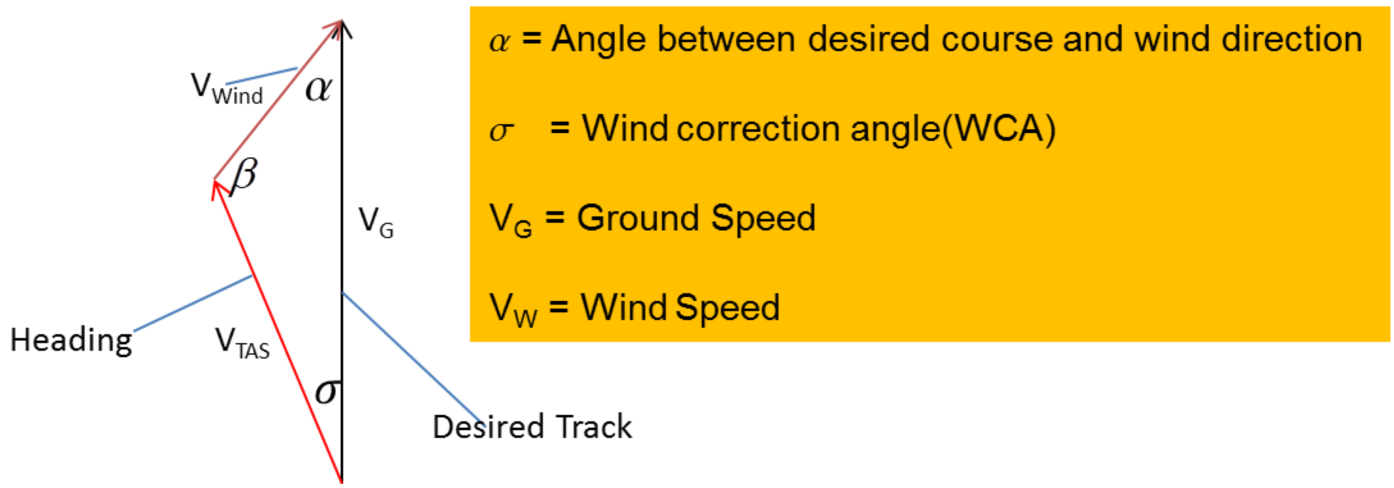

Figure 2- Basic Wind Triangle Concept

The solution to the “Wind Triangle Problem” is based on a simple concept in elementary trigonometry, called the “Law of Sine’s”. The “Law of Sine’s” can be derived using Figure 3 shown below. Since every triangle contains 3 angles which must sum to 180 degrees, we can write the following equation for the sum of the angles

$$a + b + c = 180 \qquad (1)$$

The length of the 3 sides are designated as A, B, and C. If we drop a perpendicular line from the vertex between A and B, the height, h, can be expressed

$$\frac{h}{A} = Sin(c) \qquad (2)$$

$$\frac{h}{B} = Sin(a) \qquad (3)$$

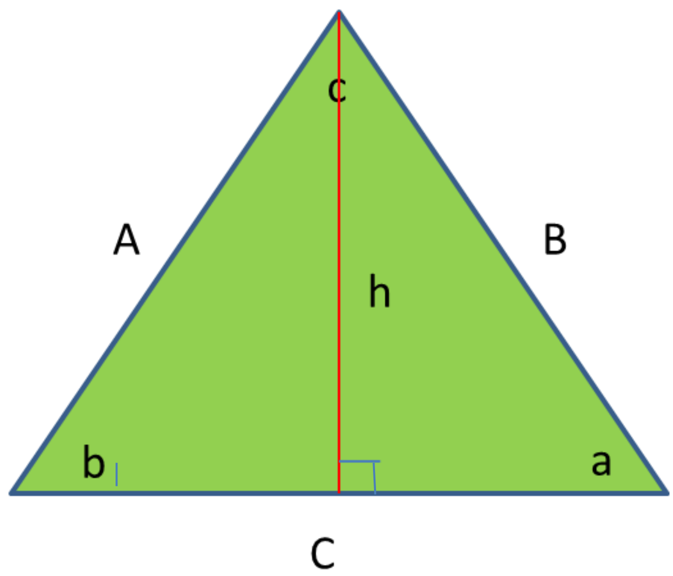

Figure 3: Concept of the "Law of Sine's"

Thus, equating the two expressions for h, we obtain the following simple relationship

$$\frac{A}{Sin(a)} = \frac{B}{Sin(b)} \qquad (4)$$

If one drops another perpendicular line from a different vertex, i.e. between A and C, one obtains a similar relationship between A and C, i.e.

$$\frac{A}{Sin(a)} = \frac{C}{Sin(c)} \qquad (5)$$

Comparing (4) and (5) we see that

$$\frac{A}{Sin(a)} = \frac{B}{Sin(b)} = \frac{C}{Sin(c)} \qquad (6)$$

Equation (6) is the fundamental relationship for the "Law of Sine's". Using (6) one can now obtain the solution of the "Wind Triangle Problem".

If we let

$$\begin{aligned} A &= V_{Wind} \\ B &= V_{TAS} \\ C &= V_G \\ a &= \sigma \\ b &= \alpha \\ c &= \beta \end{aligned} \qquad (7)$$

Equation (6) becomes

$$\frac{V_{Wind}}{Sin(\sigma)} = \frac{V_{TAS}}{Sin(\alpha)} = \frac{V_G}{Sin(\beta)} \qquad (8)$$

By equating the first two terms in (8) we obtain a simple relationship for the WCA, σ, i.e.,

$$Sin(\sigma) = (\frac{V_{Wind}}{V_{TAS}})Sin(\alpha) \qquad (9)$$

By equating the second and third terms, we obtain a relationship for the ratio of the groundspeed to the true airspeed, i.e.

$$\frac{V_G}{V_{TAS}} = \frac{Sin(\beta)}{Sin(\alpha)} \qquad (10)$$

Equation (10) contains the variable β, which is not one of the variables that the Pilot specifically inputs. Thus we need to express β in terms of α and σ. Using (1) we see that

$$\beta = 180 - \alpha - \sigma \qquad (11)$$

Therefore,

$$Sin(\beta) = Sin(180 - \alpha - \sigma) \qquad (12)$$

In order to simplify (12), we use the trigonometric relationship for the sum and difference of angles, i.e.

$$Sin(180 - \alpha - \sigma) = Sin(180)Cos(\alpha + \sigma) - \mathrm{Sin}(\alpha + \sigma)\mathrm{Cos}(180) \qquad (13)$$

Since the Sin(180)=0, and the Cos(180)=-1, (13) becomes

$$Sin(\beta) = Sin(\alpha + \sigma) \qquad (14)$$

Finally, using the trigonometric relationship for the Sine of the sum of two angles, we see that

$$Sin(\alpha + \sigma) = Sin(\alpha)Cos(\sigma) + Sin(\sigma)Cos(\alpha) \qquad (15)$$

Thus, (10) can be written as

$$\frac{V_G}{V_{TAS}} = \frac{Sin(\alpha)Cos(\sigma) + Sin(\sigma)Cos(\alpha)}{Sin(\alpha)} \qquad (16)$$

If we substitute (9) for the $Sin\sigma$, we obtain the final equation for the ratio of the groundspeed to the true airspeed, i.e.

$$\frac{V_G}{V_{TAS}} = Cos(\sigma) + (\frac{V_{Wind}}{V_{TAS}})Cos(\alpha) \qquad (17)$$

Equations (9) and (17) are the exact solution of the “Wind Triangle Problem”. The parameters that control the WCA and the groundspeed are: (1) the ratio of the wind speed to the true airspeed (V_{Wind}/V_{TAS}), and (2) The angle between the wind direction and the course to be flown (α). Equation (17) carries an extremely important piece of information that every Pilot should understand. When the angle between the wind direction and the course to be flown is 90 degrees, the Cos(90) is zero and so the groundspeed is given by $\frac{V_G}{V_{TAS}} = Cos(\sigma)$. Thus, even when the wind is 90 degrees to the course to be flown, the groundspeed will always be less than the true airspeed, since the WCA will always be greater than zero, and the $Cos\sigma$ will always be less than unity. Here, in this case (9) becomes

$$Sin(\sigma) = \frac{V_{Wind}}{V_{TAS}} \qquad (18)$$

Therefore, when the wind is blowing 90 degrees to the course to be flown, the WCA is given by the inverse Sine of the wind speed ratio. Note that when the wind speed ratio is less than a half, a good approximation for the WCA in degrees is given by

$$WCA = 60(\frac{V_{Wind}}{V_{TAS}}) \qquad (19)$$

Thus, when the wind speed ratio is 0.1, the WCA would be approximately 6 degrees. In addition, when the wind direction is less than 90 degrees to the course to be flown, the WCA will always be bounded by (19), i.e. the WCA will always be less than the value predicted by (19). We now summarize the final two equations for the solution of the "Wind Triangle Problem".

$$\begin{aligned} Sin(\sigma) &= (\frac{V_{Wind}}{V_{TAS}})Sin(\alpha) \\ \frac{V_G}{V_{TAS}} &= Cos(\sigma) + (\frac{V_{Wind}}{V_{TAS}})Cos(\alpha) \end{aligned} \qquad (20)$$

It is important that we verify that (20) is the correct solution of the "Wind Triangle Problem", by comparing the result of (20) with the solution one obtains with an Electronic E6B. We consider the following problem:

Magnetic course=360 degrees

True Airspeed=100 knots

Magnetic wind direction= 210 degrees

Wind speed=20 knots

Determine the WCA and groundspeed.

Since the wind speed ratio is 20/100=0.2, and the angle between the magnetic course and the magnetic wind direction, α is 30 degrees, the first expression in (20) provides the following

$$Sin(\sigma) = 0.2Sin(30)$$

Since the Sin(30) is 0.5, we see that $Sin\sigma = 0.1$, and thus the value of σ=5.739 degrees. The second expression in (20) requires the Cos(σ). Using the trigonometric relationship

$$Sin^2(\sigma) + Cos^2(\sigma) = 1 \qquad (21)$$

We see that Cosσ= 0.995. Therefore, the groundspeed is computed to be V_G=100[0.995+0.2Cos(30)]. Since the Cos(30) is 0.866, the groundspeed is computed to be 116.8 knots. Using the Electronic E6B Computer, we obtain the following solution for the WCA and the groundspeed.

$$\sigma = 6 \text{ degs}$$
$$V_G = 117 \text{ Kts}$$

Note that the E6B rounds off the WCA and the groundspeed, whereas the equations in (20) provide the exact unrounded results.

One can also consider the two limiting cases of a headwind and a tailwind. In the case of a tailwind, α=0 degrees, the Sin(0)=0 and the Cos(0)=1. Thus, the WCA, σ=0, and the groundspeed is just equal to the true airspeed plus the wind speed. In the headwind case, α=180 degrees, the Sin(180)=0 and Cos(180)=-1. Thus, the WCA, σ=0, and the groundspeed is just equal to the true airspeed minus the wind speed.

The important results shown in (20) will become extremely useful in discussing Turns Around a Point, which is described in Sections 4-7. Here we again show that the key parameters that affect the Pilot's ability in maintaining the constant radius during the maneuver is the wind speed ratio and the angular position of the aircraft relative to the tailwind.

In Section 3 we discuss the dynamics of the turn and the fundamental relationships between TAS, bank angle, rate of turn, and turn radius.

3.0 Dynamics of the Turn

In order to fully understand the Turn Around a Point, it is necessary for every Flight Instructor and Pilot to understand the basic concept of the dynamics of a turn. The dynamics of the turn can be determined from Newton's "Second Law of Motion", i.e. the mass of the body times the acceleration of the body is equal to the sum of all the external forces acting on the body. In the case of an airplane, the external forces are the aerodynamic forces. If all the external forces sum to zero, the aircraft is considered to be in a steady state without any acceleration. Note that acceleration of the body can arise from a rate of change in speed or direction.

When a body is travelling at a constant true airspeed in a circle of constant radius, the body has an inward radial acceleration called centripetal acceleration. This acceleration can be shown to be equal to the square of the true airspeed divided by the radius of the turn. The centripetal acceleration is due to the component of lift acting inward toward the center of the turn. If we denote inward forces and accelerations as negative and outward forces and accelerations as positive, then Newton's "Second Law of Motion" is just

$$-m\frac{V^2}{r} = -LSin(\varphi) \qquad (21)$$

Here

m= mass of the body

V=True airspeed

r= Radius of turn

L=Lift on the aircraft

ϕ= Bank angle

It is clear from (21) that as long as the lift, bank angle and true airspeed are constant, the centripetal acceleration is constant. If we transpose the centripetal acceleration to the right hand side of (21), the equation now becomes

$$0 = -LSin(\phi) + m\frac{V^2}{r} \qquad (22)$$

In this form, it appears that the aircraft is undergoing zero acceleration, i.e., the left hand side of (22) is identically zero, with the right hand side involving two forces in balance. The two forces on the right hand side of (22) are the inward component of lift and an outward force, $m\frac{V^2}{r}$ which we will call the "centrifugal force". Note that this so-called "centrifugal force" is not a physical force, but an apparent force that comes about by rewriting (21) as (22). Figure 4, taken from the Handbook of Aeronautical Knowledge, FAA-H-8083-25A shows these forces on the aircraft during a coordinated turn.

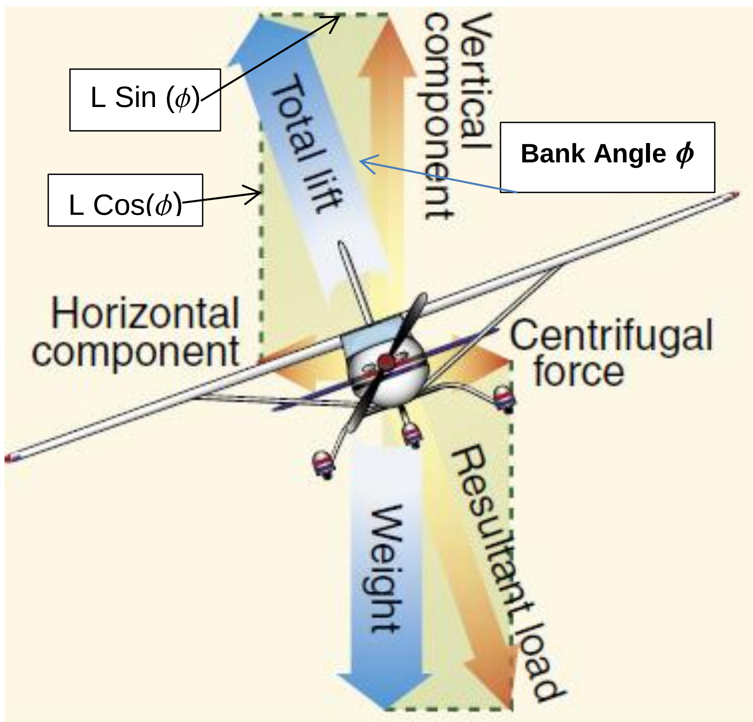

Figure 4: Forces on the Aircraft in a Coordinated Turn

If we balance the forces in the horizontal and vertical directions, we obtain the following two simple equations:

Horizontal: $$LSin(\phi) = m\frac{V^2}{r} \qquad (23)$$

Vertical: $$LCos(\phi) = W \qquad (24)$$

Dividing (23) by (24), we obtain the equation

$$Tan(\phi) = \frac{m}{W}\frac{V^2}{r} \qquad (25)$$

Since the mass of the aircraft is just equal to the weight of the aircraft divided by gravity, i.e., m=W/g, (25) becomes

$$Tan(\phi) = \frac{V^2}{gr} \qquad (26)$$

Equation (26) is the fundamental equation for the relationship between the bank angle, true airspeed and turn radius in a coordinated turn. Note that since the direction of the flight of the aircraft is always tangent to the circle, the rate of turn of the aircraft is given by the tangential speed divided by the radius of the turn, i.e.

$$ROT = \omega = \frac{V}{r} \qquad (27)$$

Equations (26) and (27) are the equations that describe the dynamics of a coordinated turn. However, if we solve for the radius from (26) and substitute it into (27), we obtain the following equation for the rate of turn:

$$ROT = \omega = \frac{gTan(\phi)}{V} \qquad (28)$$

Equation (28) shows that when the aircraft is travelling at a constant true airspeed in the turn, the ROT only depends on the bank angle ϕ. This important fact will carry over into the discussion of a Turn Around a Point.

When flying a Turn Around a Point, (26) tells us that the pilot has control over two of the three variables. Once two of the variables are specified, the third is automatically determined. As an example, let us consider an aircraft flying at various true airspeeds and bank angles, and determine what the turn radius would be for any pair of these variables. In order to determine these values, it is necessary to convert the TAS in knots to ft/sec (i.e., multiply by 1.688). In addition, the value of g is 32.17 ft/sec^2.

Table I: Turn Radius (feet) Versus KTAS/Bank Angle

	Bank Angle (degrees)		
KTAS	30	45	60
80	981	566	327
90	1242	717	414
100	1533	885	511
110	1855	1071	618

In Table 2 below, we have also tabulated the ROT in degrees/sec as a function of the true airspeed and the banks angle. Since in (28) the ROT is in radians/sec, it is necessary to multiply the result in (28) by $180/\pi$, where the value of π is 3.14159.

Table 2: ROT (degrees/sec) versus KTAS/Bank

	Bank Angle (degrees)		
KTAS	30	45	60
80	7.9	13.6	23.6
90	7.0	12.1	21.0
100	6.3	10.9	18.9
110	5.7	9.9	17.2

The information provided in Table 1 and 2 will be very useful in our discussion of the Turn Around a Point.

In Section 4 we discuss the effect of the wind on the ground track during a Turn Around a Point.

4.0 Effect of Wind on the Turn Around a Point

When an aircraft performs a Turn Around a Point without a wind, the aircraft will have a direction of flight at all times that is tangent to the circle. Equation (26) provides the radius of the turn for a given true airspeed and bank angle. In addition, there will be a corresponding rate of turn for these conditions, as shown in (27). When we attempt this maneuver in the presence of a wind, the track of the aircraft relative to the center of the circle on the ground will not be a circle. In order to understand this phenomenon, it is necessary to determine the aircraft groundspeed as a function of position around the circle. Figure 5 shows the required circle that the aircraft is attempting to track. Here we have set up an X-Y coordinate system with the center of the circle being the point (0,0).The location of the center of gravity of the aircraft along the circle is given by the angle θ measured counterclockwise from the X-axis.

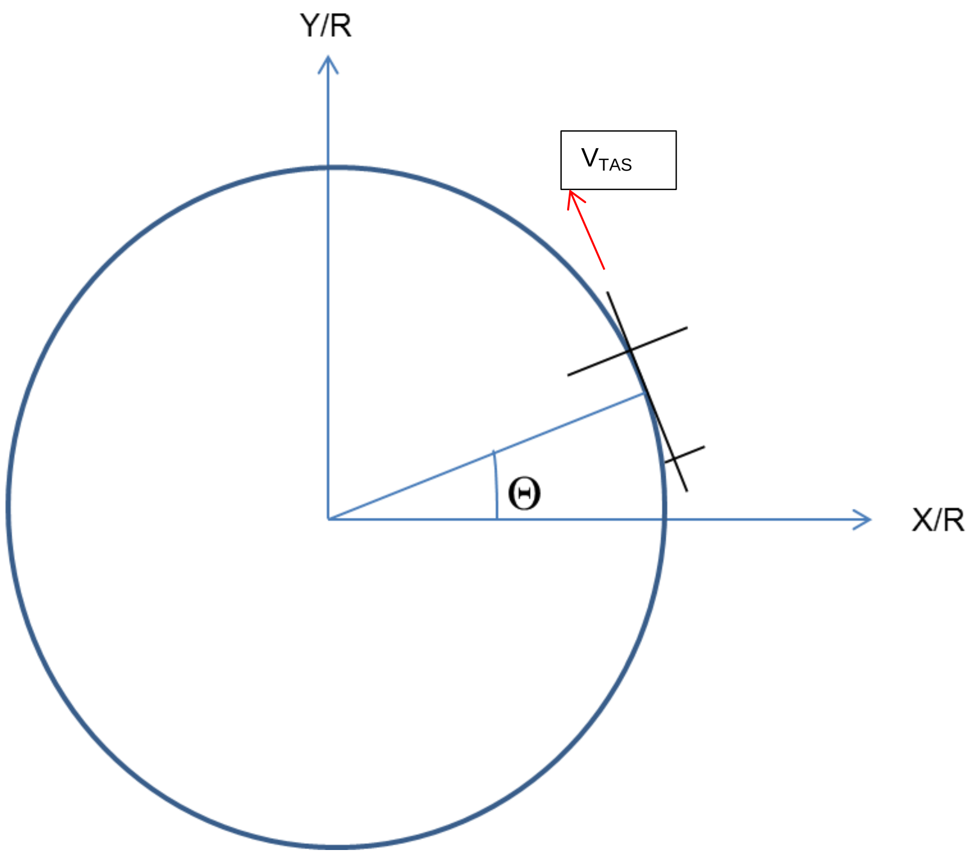

Figure 5: Coordinate System for the Turn Around a Point

We have normalized the X and Y coordinates by the radius of the circle, R. The X/R and Y/R values are given as a function of θ. Table 3 shows these coordinates corresponding to four points on the circle, i.e. θ=0, 90,180 and 270 degrees.

Table 3: X/R-Y/R Coordinates of the Circle

θ (degrees)	X/R	Y/R
0	1	0
90	0	1
180	-1	0
270	0	-1

One can express the aircraft relative heading $\Theta_{H,}$ i.e., the heading relative to the X-axis by the expression

$$\theta_H = 90 + \theta \qquad (29)$$

Thus, when θ=0, the aircraft is flying in the +Y direction. Since we can orient the wind in any direction we choose, we will allow the wind to blow from the –Y direction toward the +Y direction. Figure 5 shows the orientation of the wind relative to the Coordinate System set up. Thus, at θ=0, the aircraft is on a tailwind; at θ=90, the aircraft is on a crosswind (i.e., the downwind side), at θ=180, the aircraft on a headwind, and at θ=270, the aircraft is on the opposite crosswind (i.e., the upwind side).

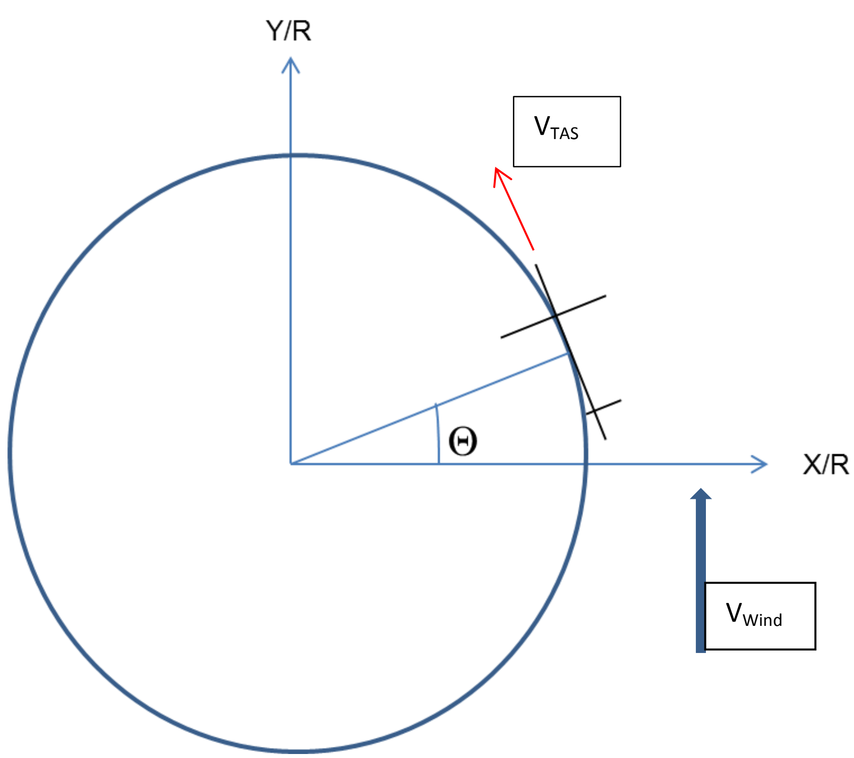

Figure 6: Wind Orientation

We now decompose the groundspeed into X and Y components.

$$\begin{aligned} V_{G_X} &= -V_{TAS} Sin(\theta) \\ V_{G_Y} &= V_{TAS} Cos(\theta) + V_{Wind} \end{aligned} \qquad (30)$$

In order for us to develop the key parameters that control the wind effect on the Turn Around A Point, we divide both sides of (30) by V_{TAS}. Equation (30) can now be written as

$$\begin{aligned} \bar{V}_{G_X} &= -Sin(\theta) \\ \bar{V}_{G_Y} &= Cos(\theta) + \bar{V}_W \end{aligned} \qquad (31)$$

Here, the overbar indicates the ground speed and wind speed have been divided by the V_{TAS}. Table 4 shows the groundspeed ratio at θ=0, 90,180 and 270 degrees.

Table 4: Groundspeed Ratio Around the Circle

θ(degrees)	V_GX	V_{GY}
0	0	$1+V_W$
90	-1	V_W
180	0	$-1+V_W$
270	1	V_W

In order to determine the actual track of the aircraft during the turn, one must perform an integration of the groundspeed over time. We normally think of distance being equal to the speed multiplied by the time. However, this is only valid when the speed is constant. As can be seen in Table 4, the groundspeed is varying around the circle. Note that when the aircraft is turning at some given rate of turn, ω, the time interval can be expressed as the change in θ divided by the rate of turn, i.e.

$$dt = \frac{d\theta}{\omega} \qquad (32)$$

Recalling that the rate of turn is just $\frac{V_{TAS}}{R}$, we see that we can rewrite (32) as

$$dt = \frac{Rd\theta}{V_{TAS}} \qquad (33)$$

Therefore, the X and Y coordinates of the aircraft track can now be obtained by integrating the groundspeed over θ, between θ=0, and any arbitrary value of θ. The resulting expression for the normalized coordinates are

$$\begin{aligned} \frac{X}{R} &= \int_0^{\theta_f} [-Sin(\theta)]d\theta + 1 \\ \frac{Y}{R} &= \int_0^{\theta_f} [Cos(\theta) + \bar{V}_W]d\theta \end{aligned} \qquad (33)$$

Here $\theta_{f,}$ is the value of θ at the location we require X/R and Y/R. Elementary calculus provides us with the following values of X/R and Y/R:

$$\begin{aligned} \frac{X}{R} &= Cos(\theta_f) \\ \frac{Y}{R} &= Sin(\theta_f) + \bar{V}_W \theta_f \end{aligned} \qquad (34)$$

Equation (34) tells us that after making a 360 degree turn at a constant bank angle, the values of X/R and Y/R are given by

$$\begin{aligned} \frac{X}{R} &= 1 \\ \frac{Y}{R} &= 2\pi \bar{V}_W \end{aligned} \qquad (35)$$

Therefore, after making a 360 degree turn at a constant bank angle, the aircraft will return to the same X/R location, but will be shifted downwind (+Y-direction) by the amount $\frac{Y}{R} = 2\pi \bar{V}_W$. Note that $\frac{2\pi R}{V_{TAS}}$ is exactly the time it take the aircraft to turn 360 degrees (i.e.2π radians). Figure 7 shows the aircraft track at a constant bank angle, corresponding to wind speed ratios of 0, 0.1, 0.2, and 0.3. It is easy to see that at the end of the 360 degree turn, the Y/R values corresponding to V_W =0, 0.1, 0.2, and 0.3 are just $0,\ 0.2\pi,\ 0.4\pi,$ and 0.6π.

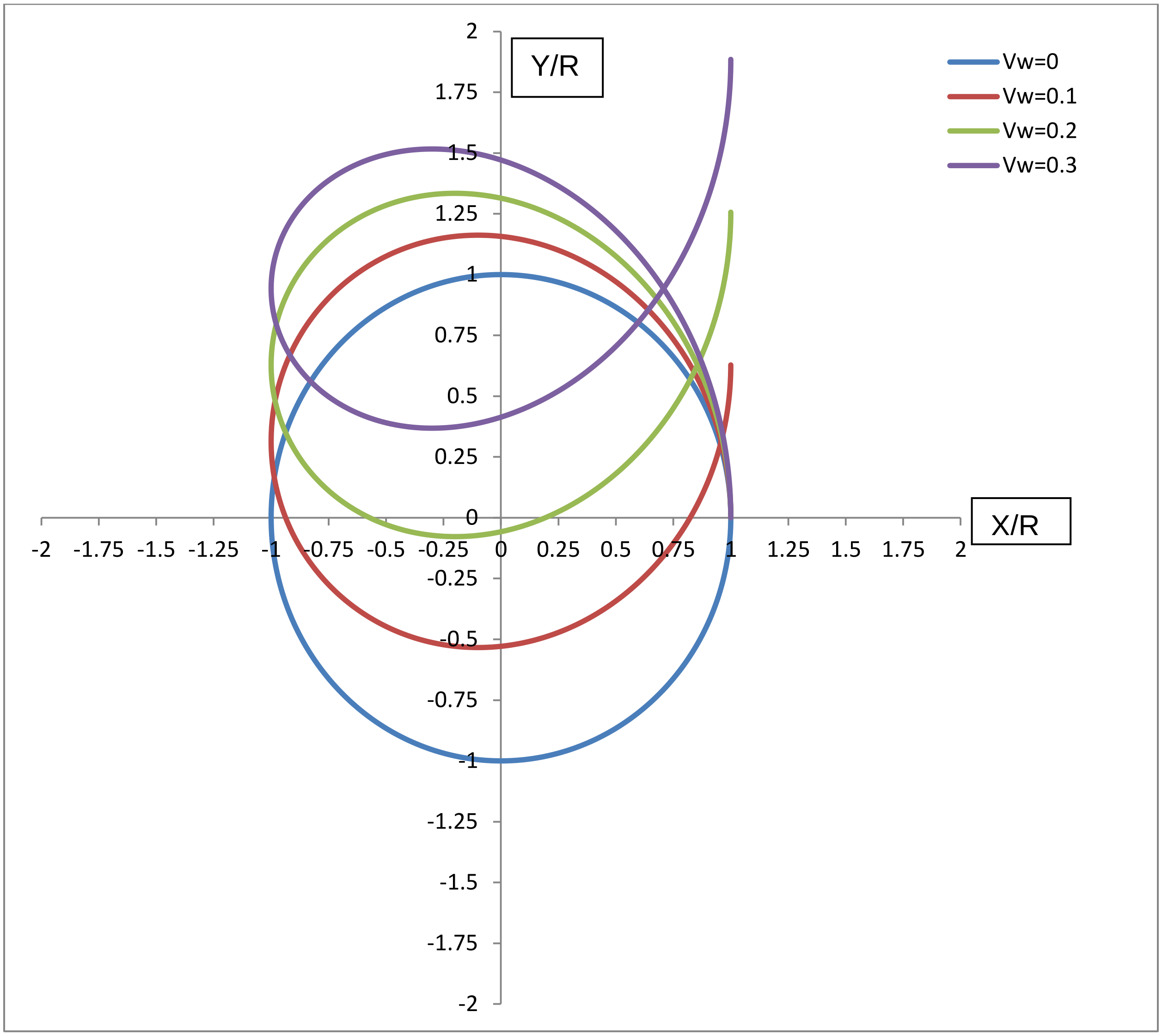

Figure 7: Aircraft Track for a 360 Degree Turn at Constant Bank Angle

Wind Speed Ratios 0, 0.1, 0.2 and 0.3

Figure 7 clearly shows that a constant bank angle will not allow the aircraft to track a constant radius circle. However, it is important that all Pilots understand the following concept: Consider at time zero two observers are situated right over the center of the circle to be flown. The first observer is stationary, and the second observer is travelling with the velocity of the air mass (i.e. the wind). The stationary observer will see the aircraft track as shown in Figure 7. However, the observer moving with the air mass velocity (i.e. the wind speed and direction) will see the aircraft executing a perfect circle. The radius of this circle will depend on the true airspeed and the bank angle flown, and will be determined from (26).

In Section 5 we show how the solution of the "Wind Triangle Problem" can be used to develop the exact solution to the Turn Around a Point maneuver.

5.0 Exact Solution to the Turn Around a Point

In order to determine the required rate of turn to perform a Turn Around a Point in the presence of a wind, it is necessary to relate the "Wind Triangle Problem" to the Turn Around a Point maneuver. Let us start off with a circle of a given radius R, and then divide it into 8 equal-angle sectors (i.e. 45 degrees each). Figure 8 shows a schematic of this operation.

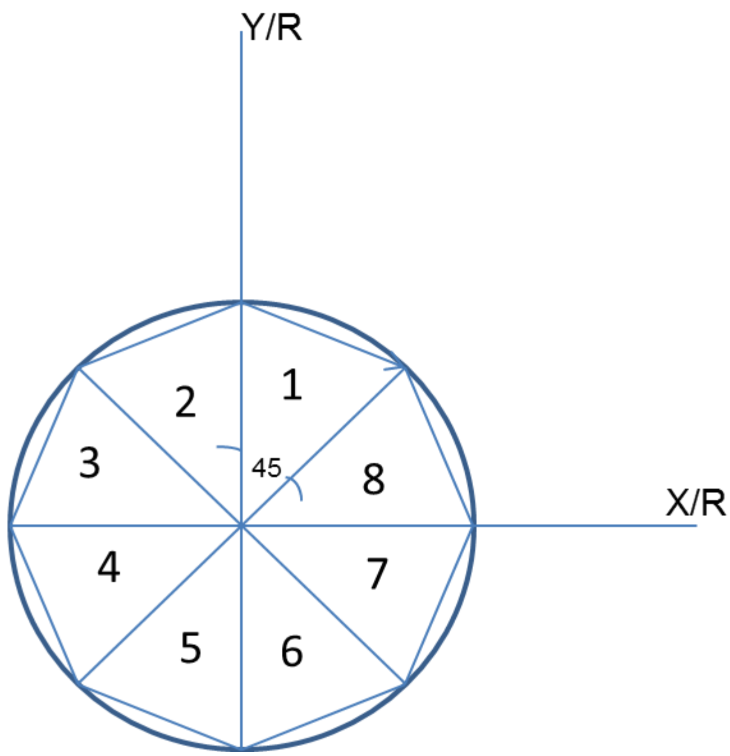

Figure 8: Schematic Showing the Division of the Circle into 8 Equal-Angle Sectors

We now connect each of the sectors with straight lines, such that the beginning and ending point of each straight line lies on the circle. If we consider each of these straight lines as a road, it becomes clear that if one starts on the circle at the beginning of the road and then tracks along the road, the aircraft will end up at the endpoint of the road, which is also on the circle. Thus, it is easy to see that one needs to determine the wind correction angle along each of these roads in order to arrive at the endpoint on the circle. Note that except for the beginning and endpoints, these roads in Figure 8 do not lie on the circle. One can now consider the scenario of tracking a road from the beginning to the endpoint and then turning to the next road and tracking that road, continuing all around the circle. Each road will have associated with it, a wind correction angle and a groundspeed. At the end of each road the aircraft will need to turn to a new heading which corresponds to the heading of the new road plus the wind correction angle for the new road. Since all the roads are the same distance, the higher the groundspeed, the shorter the time necessary to establish the correct heading change. Therefore, there are two parameters that affect the required rate of turn to stay on the circle. The first is the groundspeed, which controls the amount of time the aircraft has to change the heading; and second, the amount of heading change due to the differences in both the wind correction angles, and the change in course direction of the two consecutive roads. If we now consider increasing the number of sectors more and more, it is easy to see that the length of these roads becomes shorter and shorter and eventually the roads will be orientated tangent to the circle. In this limiting process, the track of the aircraft will lie exactly on the circle.

With the above process in mind, we can now develop a simple expression for the required rate of turn as a function of the angular position around the circle. Figure 2 showed the set-up for the solution of the “Wind Triangle Problem”. Note that the angle α is the relative wind angle measured from the tail of the aircraft. Figure 6 shows the orientation of the wind relative to the X-Y Coordinate System. It is easy to see that the angle α is the same as the angle θ in Figure 6. Thus, (20) can simply be rewritten as

$$\begin{aligned} &Sin(\sigma) = \bar{V}_W Sin(\theta) \\ &\frac{V_G}{V_{TAS}} = \bar{V}_G = Cos(\sigma) + \bar{V}_W Cos(\theta) \end{aligned} \qquad (36)$$

We can express the required heading of the aircraft relative to the X-axis in terms of both θ and σ, i.e.

$$\theta_H = 90 + \theta + \sigma \qquad (37)$$

Since we are interested in the rate of turn of the aircraft, we need the rate of change of θ_H with respect to time. This can be expressed as

$$\frac{d\theta_H}{dt} = \frac{d\theta}{dt} + \frac{d\sigma}{dt} \qquad (38)$$

The first term on the right hand side corresponds to the change in the angle of the road between two consecutive sectors, and is given by

$$\frac{d\theta}{dt} = \frac{V_G}{R} = \frac{V_{TAS}}{R}\bar{V}_G \qquad (39)$$

The second term requires the time rate of change of the wind correction angle. From (36) we see that

$$\sigma = Sin^{-1}[\bar{V}_W Sin(\theta)] \qquad (40)$$

We can express the rate of change of the wind correction angle with respect to time as

$$\frac{d\sigma}{dt} = \frac{d\sigma}{d\theta}\frac{d\theta}{dt} \qquad (41)$$

Substituting (39) and (41) into (38) we obtain the following expression for the rate of change of the heading

$$\frac{d\theta_H}{dt} = \frac{V_{TAS}}{R}\bar{V}_G(1 + \frac{d\sigma}{d\theta}) \qquad (42)$$

Recall that the rate of turn of the aircraft is given by (27). Thus, the required rate of turn that one would need in order for the aircraft to track the circle without a wind is $\frac{V_{TAS}}{R}$.

Therefore, we can now express the ratio of the required turn rate with a wind, to that without a wind as

$$\Omega = \frac{\frac{d\theta_H}{dt}}{(\frac{V_{TAS}}{R})} = \bar{V}_G(1 + \frac{d\sigma}{d\theta}) \qquad (43)$$

In order to obtain Ω as a function of the angular position around the circle we need to obtain $\frac{d\sigma}{d\theta}$. We can differentiate (40) to obtain this derivative. The process of differentiation is just finding the slope of any function. Using elementary calculus we find

$$\frac{d\sigma}{d\theta} = \frac{\bar{V}_W Cos(\theta)}{\sqrt{1 - \bar{V}_W^2 Sin^2(\theta)}} = \frac{\bar{V}_W Cos(\theta)}{Cos(\sigma)} \qquad (44)$$

Substituting (44) into (43), and using (36) for the definition of the groundspeed, we arrive at the final result for Ω, i.e.

$$\Omega = \frac{\bar{V}_G^2}{Cos(\sigma)} \qquad (45)$$

Equations (36) and (45) provide the exact solution to the Turn Around A Point maneuver. It provides the wind correction angle, groundspeed, and turn rate as a function of the angular position measured from the tailwind position (θ=0). The solution is in terms of the wind speed ratio, $\bar{V}_W$ .Thus, once the wind speed ratio is specified, the entire solution to the Turn Around a Point is known. Although (45) provides the rate of turn, from a Pilot's standpoint, it is best to express this in terms of the bank angle as a function of angular position around the circle. Using (28) we can rewrite (45) as

$$\frac{Tan(\phi)}{Tan(\phi_0)} = \frac{\bar{V}_G^2}{Cos(\sigma)} \qquad (46)$$

Where the required bank angle in the no-wind case is ϕ_0, and ϕ is the required bank angle for a finite wind speed ratio. Note that the no-wind bank angle is used because the actual radius of the turn can be related to the no-wind bank angle and the TAS (i.e. (26)).The information contained in (45) and (46) is extremely important in understanding how to execute a precise Turn Around a Point. In order to show the importance of these results, we will consider wind speed ratios ranging from 0 to 0.3. For typical General Aviation aircraft executing a Turn Around a Point, a wind speed ratio of 0.3 would correspond to a wind speed of 25-35 knots. Note that when $\bar{V}_W = 0$ the groundspeed ratio, $\bar{V}_G = 1$ and the wind correction angle is identically zero. Thus Ω=1, and $\varphi = \varphi_0$.

Figure 9 shows the required rate of turn ratio Ω versus θ for wind speed ratios corresponding to 0.1, 0.2, and 0.3.

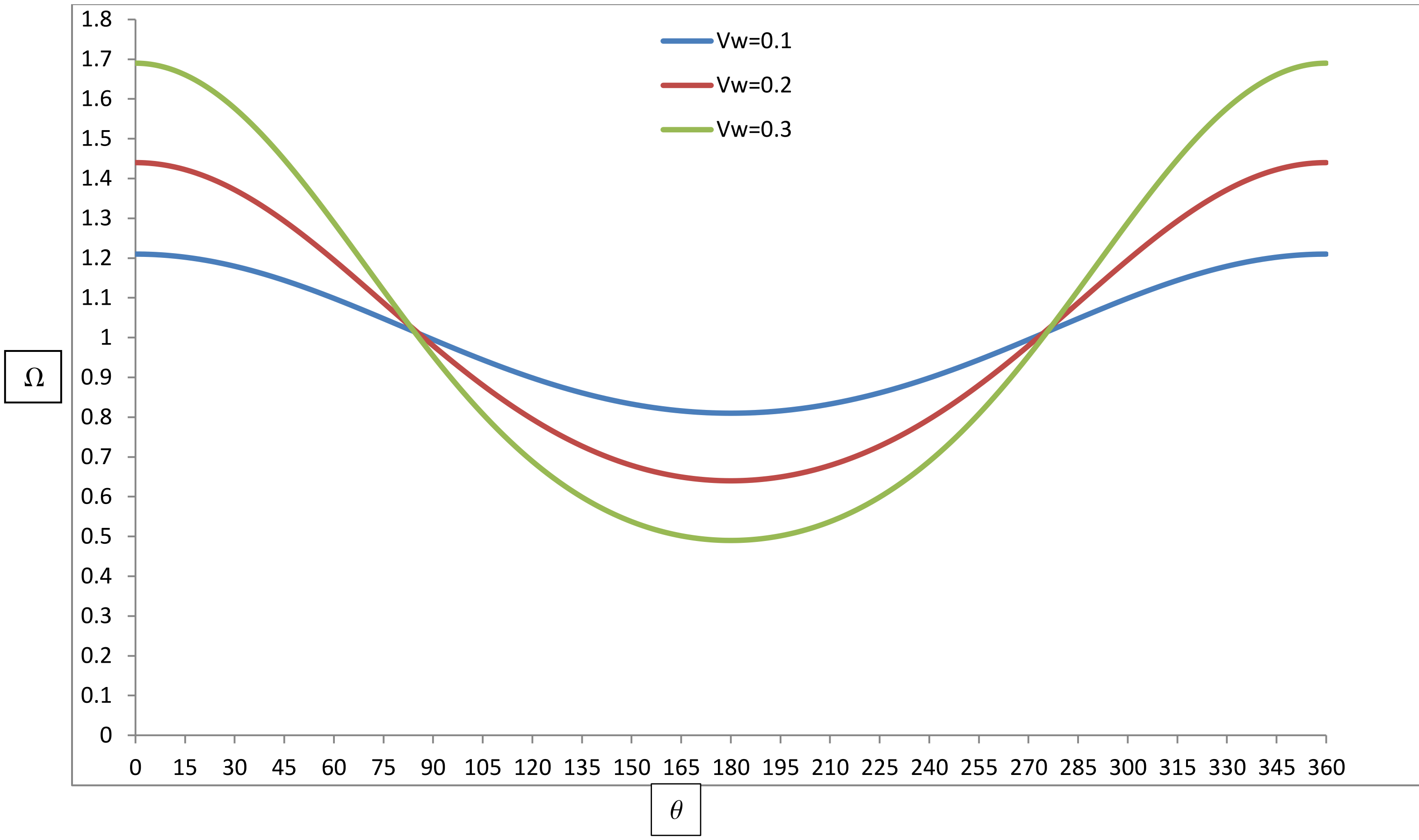

Figure 9: Required Turn Rate Ratio Versus θ

This Figure shows a number of important elements that Pilots need to be aware of in performing a precise Turn Around a Point.

(a) Maximum turn rate is on the tailwind
(b) Minimum turn rate is on the headwind
(c) The turn rate is symmetric around the line connecting θ=0 and 180 degrees
(d) The rate of change of the turn rate is nearly linear in regions away from the headwind or tailwind. This rate of change in the required turn rate is proportional to the wind speed ratio.

(e) In the neighborhood of the headwind or tailwind, the turn rate does not change much. This region near the headwind/tailwind becomes narrower as the wind speed ratio increases.

(f) For wind speed ratios less than or equal to 0.3, the maximum wind correction angle is less than 20 degrees and thus Cosσ can be approximated by unity (i.e. within about 6 %). Therefore, the turning rate ratio can be approximated by $\Omega \simeq \bar{V}_G^2$ and the turning rate is dominated by the groundspeed ratio.

(g) The position for which the turning rate ratio is unity does not correspond to the 90 degree (crosswind/downwind side) or the 270 degree (crosswind/upwind side) locations. This is because the groundspeed ratio is less than unity at the 90/270 degrees points. The location for Ω=1 will occur slightly before 90 degrees and slightly after 270 degrees (i.e. on the tailwind side of the circle), the exact location depending on the wind speed ratio.

From a Pilot's standpoint in flying the maneuver, the turn rate is not as desirable a visual cue as the bank angle when performing ground reference maneuvers. Equation (46) provides the bank angle ratio as a function of θ. Figure 10 shows the required bank angle as a function of θ. Here we have assumed a no-wind bank angle of 45 degrees.

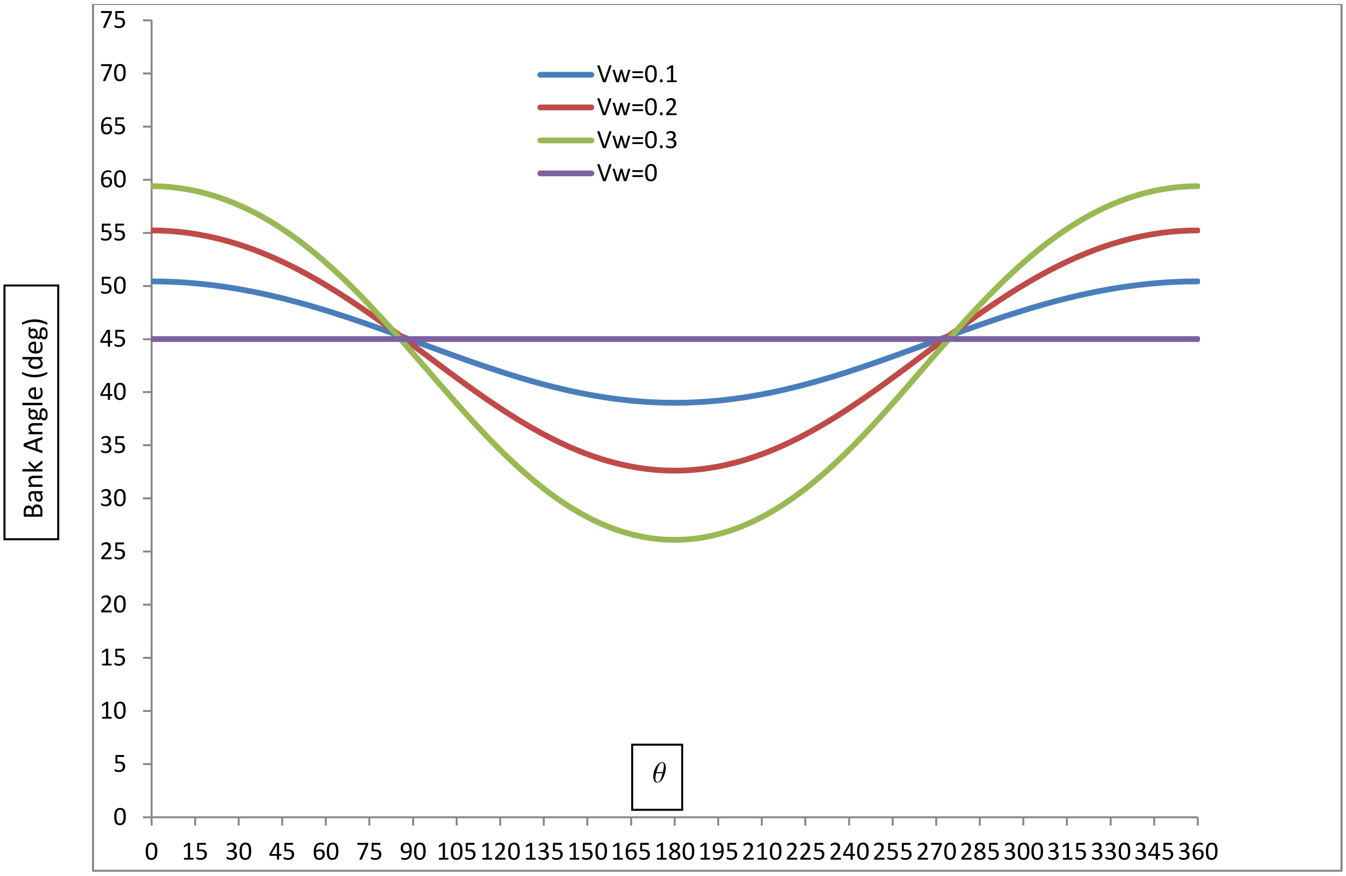

Figure 10: Required Bank Angle Versus θ for a Turn Around A Point

This Figure provides critical information for the pilot in performing a precise Turn Around a Point. These key points are described below.

(a) The maximum bank angle occurs on the tailwind.
(b) The minimum bank angle on the headwind
(c) The bank angle is symmetric around the line connecting θ=0 and 180 degrees
(d) Compared to the no-wind bank angle, there is a larger decrease in bank angle on the headwind than there is an increase on the tailwind. As an example, for V_W=0.3, the bank angle on the tailwind increases from 45 to 60 degrees (15 degrees), whereas, on the headwind, the bank angle decreases from 45 to 25 degrees (20 degrees).
(e) In the narrow region around the tailwind/headwind, the bank angle is nearly constant, whereas, at all other points around the circle, the bank angle is either continually increasing or continually decreasing.

An important parameter in executing the maneuver is the “gradient bank angle”, which is defined as the required change in bank angle per degree of change in position around the circle. It will be shown in the next Section that the "gradient bank angle" is proportional to the wind speed ratio, and is an extremely important parameter for executing the maneuver with precision. In addition, the higher the wind speed ratio, the narrow the region around the tailwind/headwind location where the bank angle remains constant.

Figure 11 shows the wind correction angle versus angular position around the circle.

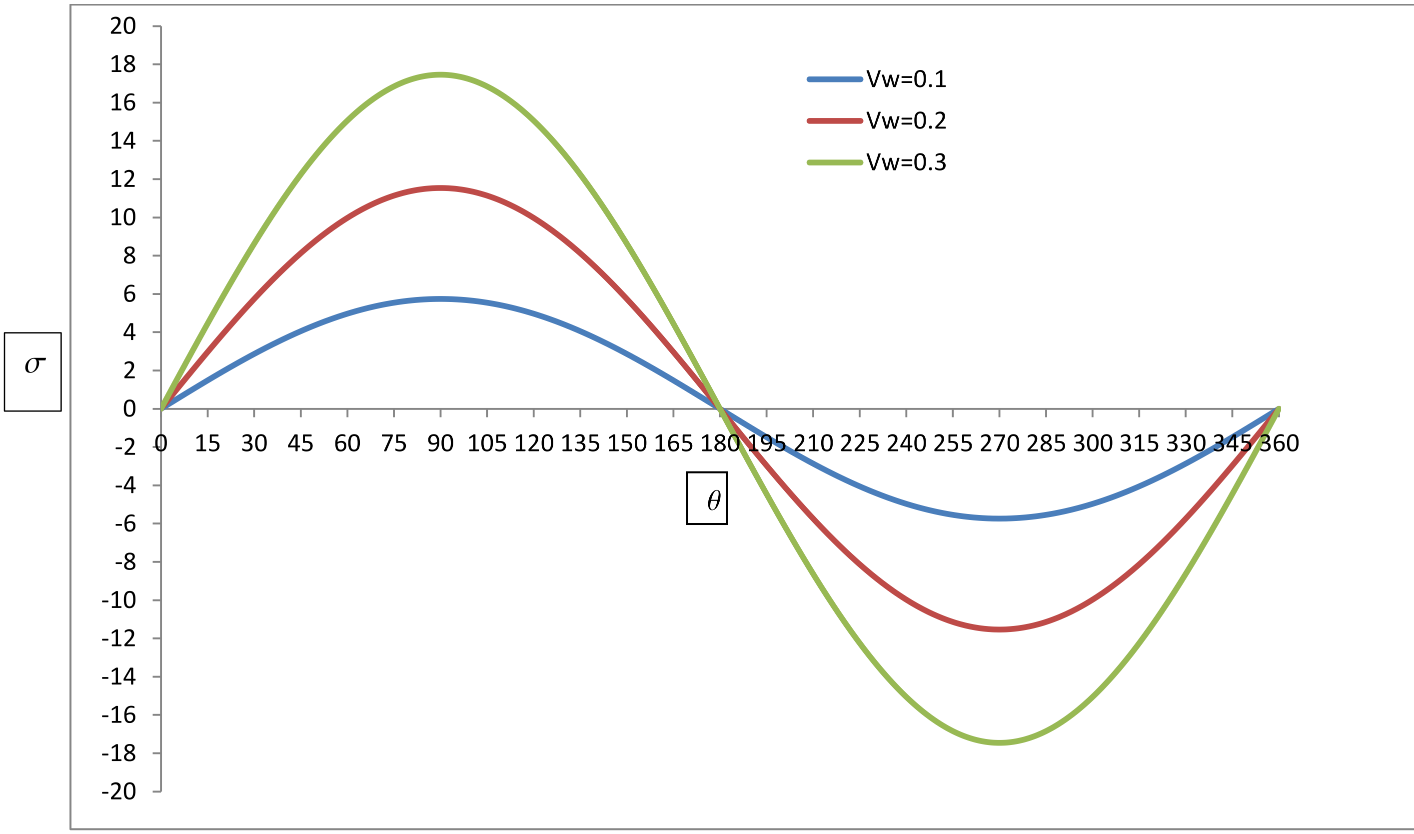

Figure 11: Wind Correction Angle Versus θ

Note that the wind correction angle is anti-symmetric around the line connecting θ=0 and 180 degrees, with the positive values requiring the aircraft to be crabbed in toward the center of the circle, and the negative values requiring the aircraft to be crabbed outward away from the center of the circle.

Figure 12 shows the groundspeed ratio as a function of the angular position around the circle for the three wind speed ratios of interest.

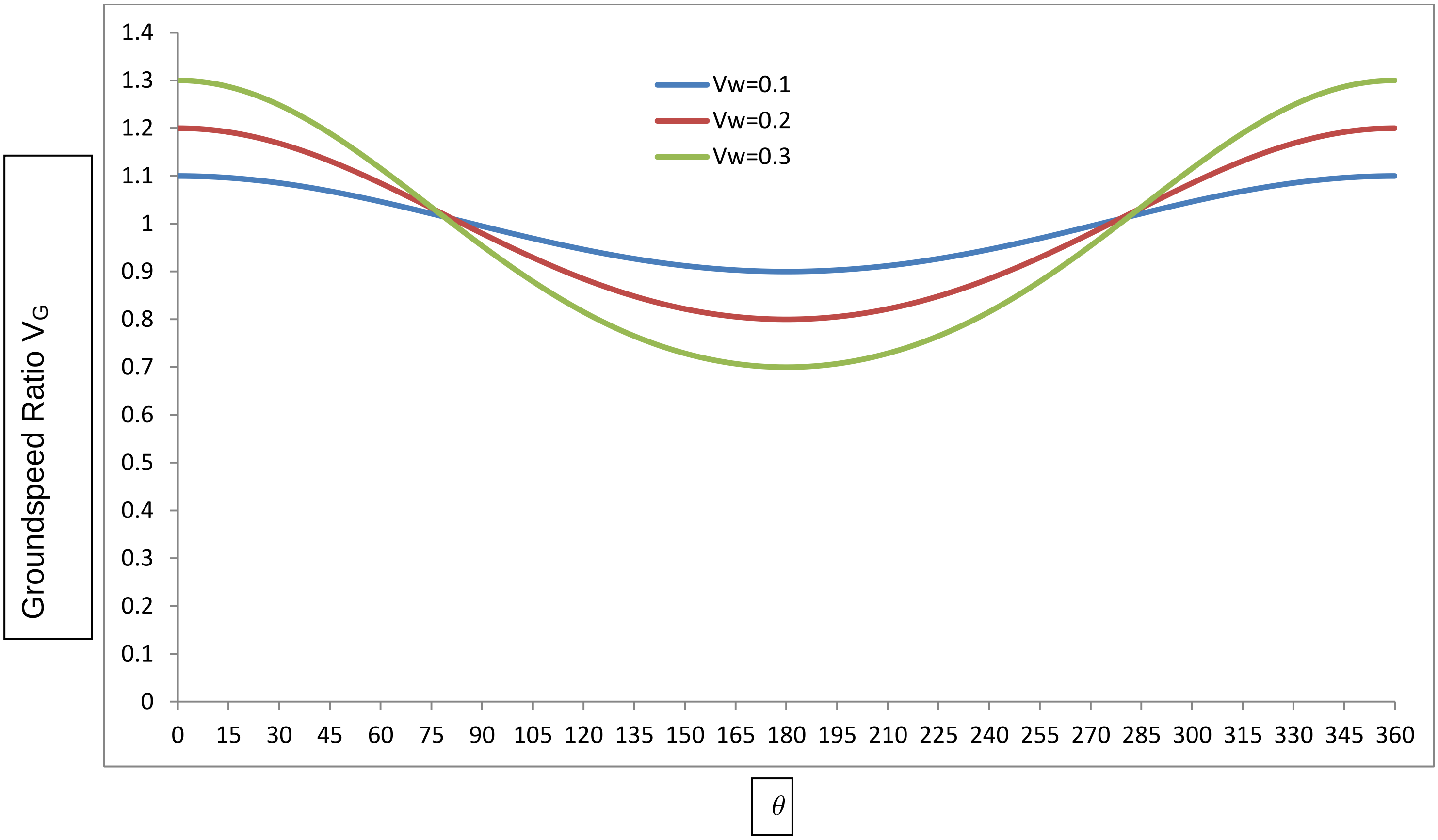

Figure 12: Groundspeed Ratio Versus θ

As can be seen in Figure 12, the groundspeed ratio has the same character as both the required turn rate and the bank angle shown in Figures 9 and 10.

In Section 6 we discuss the fine points of executing a precision Turn Around a Point.

6.0 Flying the Turn Around a Point

The analysis in Section 5 provides a considerable amount of information that tells the Pilot exactly how to fly the maneuver. Clearly, when there is no wind, the Turn Around A Point maneuver is flown at constant airspeed at a given angle of bank corresponding to the radius of the circle to be flown. Although the aircraft is flown at a particular IAS for the maneuver, it is important to understand the true airspeed is what we are concerned about when performing the maneuver. For example, Table 1 shows that at 90 KTAS, the turn radius for a 45 degree bank angle is 717 feet, and at 100 KTAS, the turn radius for the same bank angle is 885 feet. Thus, it is clear that one important part of training Student

Pilots to perform this maneuver is their ability to estimate distances from a point on the surface when flying at low altitudes. One can use a couple of different methods to help the Student become proficient in this task. The first method is to use the stripes on the runway when flying in the pattern. The distance from the beginning of any stripe to the beginning of the next stripe is 200 feet. Thus 4 stripes amount to a distance of 800 feet, which is close to the 717 feet for the 45 degree bank Turn Around a Point. In addition, one can fly at a height above the ground equal to the required radius, and then project a 45 degree line of sight from the horizontal, downward toward the surface. The intersection of the line of sight with the surface gives the radius from that point equal to the altitude the aircraft is flying over the ground. Although not exact, the method should be able to provide a value of the radius that is relatively close to the radius the Pilot is planning on using.

When performing the maneuver without a wind, Table 1 should provide a very good first approximation to the required bank angle. However, since the radius is approximated, the Pilot may be required to change the bank angle slightly until a fixed bank angle is found that holds that exact radius.

When we introduce the wind into the Turn Around A Point, the maneuver becomes significantly more complex. If one reads FAA-H-8083-3A, the FAA "Flight Training Handbook", Chapter 4, the FAA recommends that if the Pilot wishes to use a maximum bank angle of 45 degrees," the aircraft should enter the maneuver initially with a 45 degree bank on the tailwind, if the aircraft is at the correct distance from the point". However, there is no quantitative information provided as to what that radius should be. Note that for a given TAS, the required radius will be a function of the wind speed ratio. Figure 13 shows the required bank angle during a Turn Around a Point corresponding to a no-wind bank angle of 35 degrees. It is clear that for a wind speed ratio of 0.1, the required bank angle on the tailwind (θ=0) is 40 degrees, whereas, for a wind speed ratio of 0.2, the required bank angle on the tailwind is 45 degrees. Since the no-wind bank angle is 35 degrees, the radius of the turn corresponding to a TAS of 90 knots, can be determined from (26), and is 1024 feet.

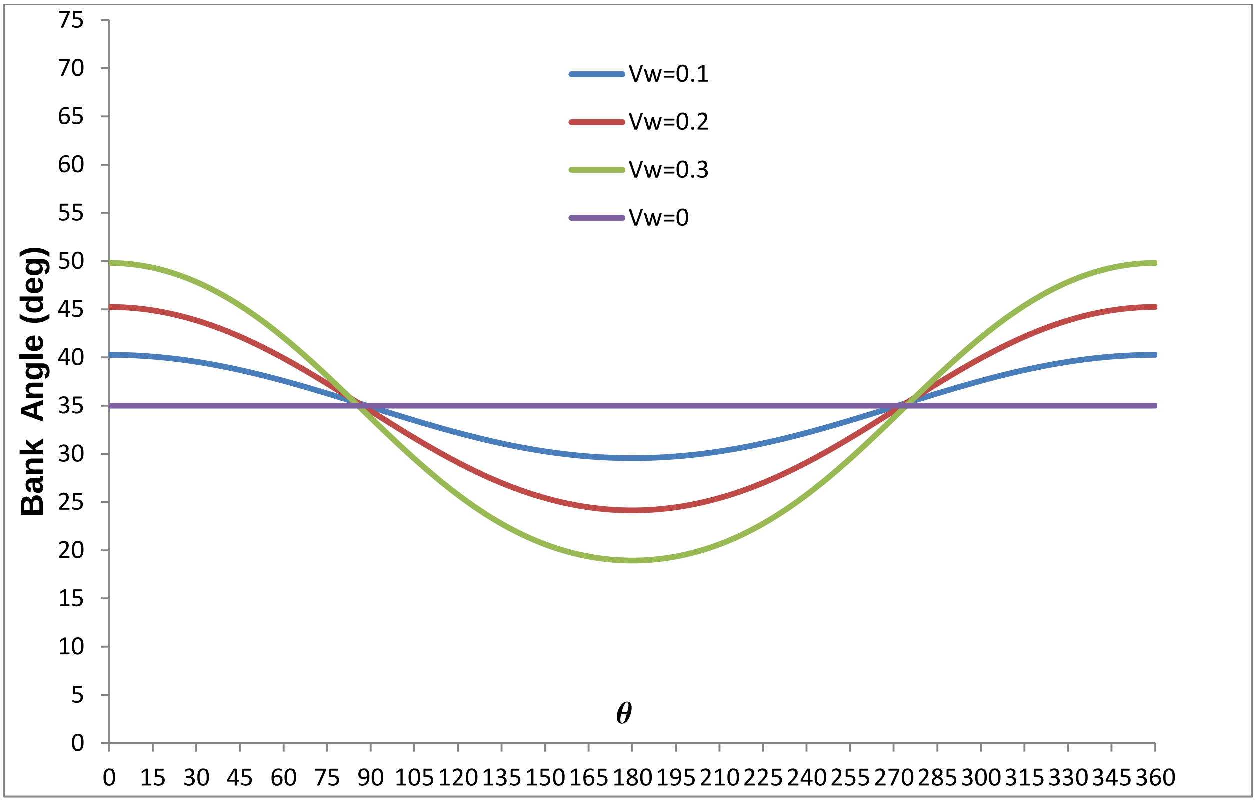

Figure 13: Required Bank Angle versus θ Corresponding to a 35 degree No-Wind Bank Angle

Now, let us consider initiating the Turn Around a Point at a radius of 717 feet from the center point. This corresponds to a no-wind bank angle of 45 degrees at 90 KTAS. One can clearly see from Figure 10 that with a wind speed ratio of 0.1 (9 knots), the required bank angle on the tailwind would be 50 degrees, and with a wind speed ratio of 0.2 (18 knots), the required bank angle on the tailwind would be 55 degrees. Although Figures 10 and 13 can be used to bracket the no-wind bank angle, we can determine the no-wind bank angle directly from equation (46). Note that on the downwind, the wind correction angle σ=0, and thus, Cos(0)=1. Since the maximum bank angle (Φ) is to be 45 degrees, Tan (45) =1, and (46) reduces to the simple expression for the no-wind bank angle, i.e.

$$Tan(\phi_0) = \frac{1}{(1+\bar{V}_W)^2} \qquad (47)$$

As can be seen in (47), when the wind speed ratio is zero, the no-wind bank angle is 45 degrees (i.e. $\phi_0 = Tan^{-1}(1) = 45$ degrees). Note that for $\bar{V}_W = 0.2$, the no-wind bank angle would be 34.8 degrees, which corresponds closely to the results shown in Figure 13.

In order not to exceed the maximum bank angle of 45 degrees on the downwind, the Pilot needs to know the required radius of the turn. Since equation (26) provides the turn radius as a function of the bank angle and the true airspeed, it is easy to see that the required turn radius with a wind which keeps the maximum bank angle at 45 degrees, is given by

$$r = r_0(1+\bar{V}_W)^2 \qquad (48)$$

Here, r_0 is the required no-wind radius, and r is the required radius corresponding to a wind speed ratio of $\bar{V}_W$. Note that the required no-wind radius corresponding to a 45 degree bank angle is given by

$$r_0 = \frac{V^2}{g} \qquad (49)$$

It is easy to show from the above equations that if the Pilot chose a no-wind bank angle of 45 degrees (i.e. Figure 10), the maximum bank angle that would occur on the downwind would be given by

$$Tan(\phi) = (1+\bar{V}_W)^2 \qquad (50)$$

For example, in the case of $\bar{V}_W = 0.3$, the value of Φ=59.4 degrees, which corresponds to the value shown in Figure 10 (i.e., the green line).

Figure 13 also provides an additional key piece of information that all Pilots should be aware of: On the downwind side (θ=90), and on the upwind side (θ=270), the required bank angle would be close to the no-wind bank angle, i.e. 35 degrees. One needs to understand that the reason it is not exactly 35 degrees is that the groundspeed at these two points is not equal to the true airspeed, due to the wind correction angles established at those points.

We should point out that in all the current textbooks which discuss how to fly the Turn Around a Point, what is emphasized is: (a) The maximum bank is on the tailwind, (b) The minimum bank is on the headwind. Thus the bank angle must be increasing from the point when the aircraft is on the headwind, to the point when the aircraft is on the tailwind; and the bank angle must be decreasing from the point when the aircraft is on the tailwind, to the point when the aircraft is on the headwind. Therefore, it is left up to the Flight Instructor/Pilot to figure out the missing details. These missing details will be discussed below.

The only two quantities of interest to the pilot when flying a Turn Around a Point are: (a) The bank angle as a function of angular position around the circle, and (b) The “gradient bank angle” as a function of the angular position around the circle. Since Figure 13 shows the required bank angle as a function of the angular position around the circle, the “gradient bank angle” is just the slope of this curve as a function of the angular position the circle. Since (46) provides the bank angle as a function of θ, one can differentiate this expression to obtain the slope of the curve. The “gradient bank angle” parameter is given below

$$\frac{d\phi}{d\theta}=\frac{Tan(\phi_0)\bar{V}_W\bar{V}_G[\bar{V}_G Tan(\sigma)Cos(\theta)-2Sin(\theta+\sigma)]}{Cos^2(\sigma)[1+\Omega^2 Tan^2(\phi_0)]} \tag{51}$$

Note that the “gradient bank angle” is proportional to the wind speed ratio, so that increasing the wind speed ratio by a factor of 2 will increase the required “gradient bank angle” by a factor of 2. In addition, it is easy to see that the “gradient bank angle” is anti-symmetric around the line connecting θ=0 and180 degrees.

We can now evaluate the “gradient bank angle” at points 1-4 around the circle. At θ= 0 and θ=180, the wind correction angle, σ is zero, and therefore $\frac{d\phi}{d\theta}=0$. When θ=90 or θ=270, the “gradient bank angle” is given by

$$\frac{d\phi}{d\theta}=\mp\frac{2\bar{V}_W Tan(\phi_0)}{[1+(1-\bar{V}_W{}^2)Tan^2(\phi_0)]} \tag{52}$$

Where the minus sign corresponds to θ=90, and the plus sign corresponds to θ=270. Figure 14 shows the “gradient bank angle” for a no-wind bank angle of 35 degrees, corresponding to the three wind speed ratios of interest.

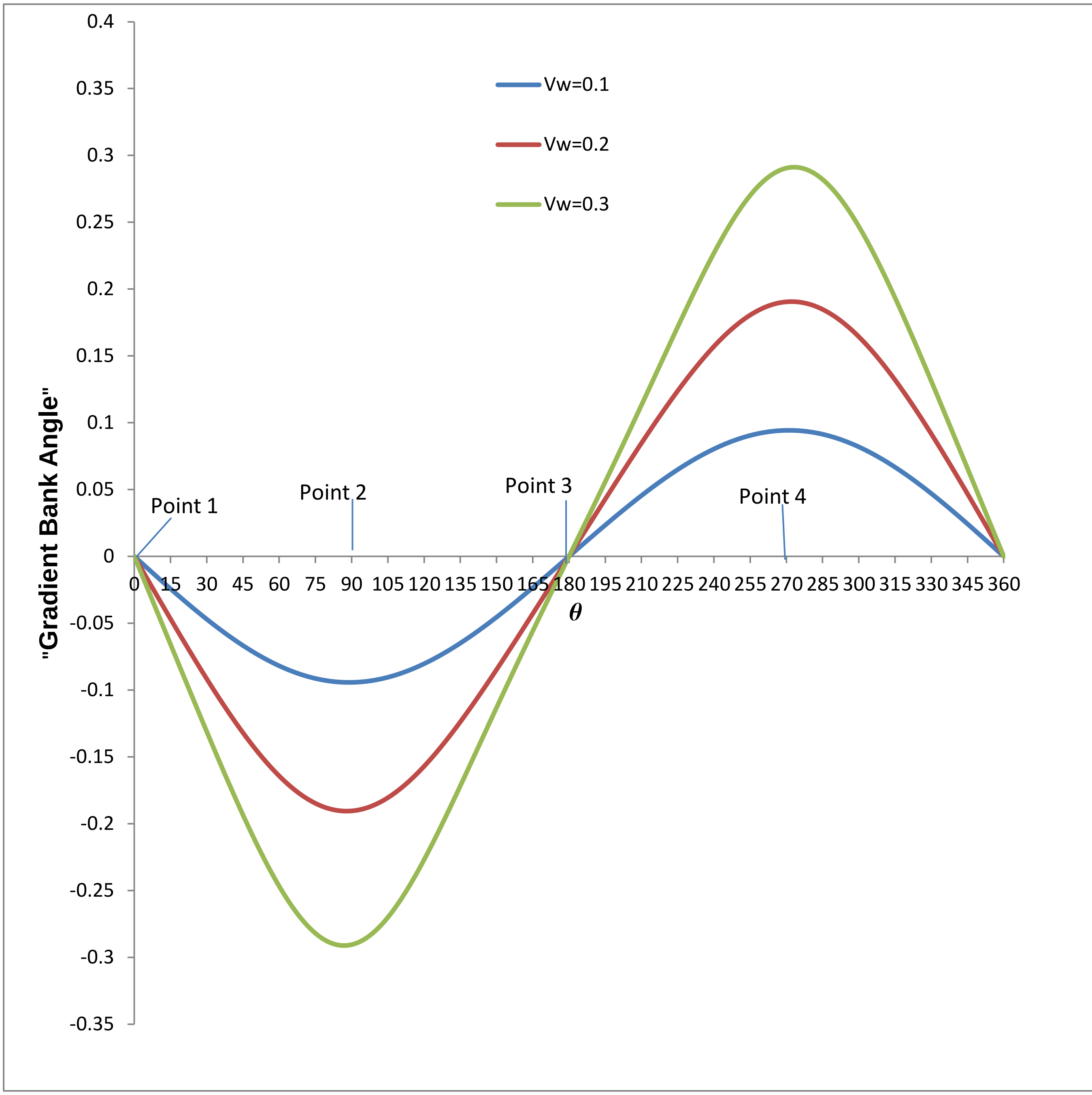

Figure 14: "Gradient Bank Angle" Versus θ for Various Wind Speed Ratios

No-Wind Bank Angle 35 Degrees

This figure provides the Pilot with the most important information on how to fly a precision Turn Around a Point. The maximum "gradient bank angle", which occurs near θ=90 and θ=270, is only 0.3. This means that for every 10 degrees around the circle, the bank angle will change no more than 3 degrees. At other points around the circle the required change in bank angle every 10 degrees is less than this value. In fact on the tailwind (point 1) and the headwind (point 3), the “gradient bank angle” is identically zero, which means the bank angle is held constant for a short period of time when near these points. Thus, even with a wind speed ratio of 0.3, which when flying the maneuver at a TAS of 90 knots, corresponds to a 27 knot wind, the maneuver is still flown with very small bank angle increases or decreases as the aircraft proceeds around the circle.

If we follow the aircraft around the circle, starting from the tailwind position, we can summarize the execution of the maneuver below:

(1) The aircraft enters with the maximum bank angle at the tailwind point. The bank angle is held fairly constant for a short angular distance in travelling from point 1 to point 2.
(2) The bank angle starts to decrease first at a slow rate, and then at a faster rate, until the aircraft is near point 2. Note that this faster rate would be no more than 3 degrees for every 10 degrees of angular movement around the circle.
(3) After passing point 2, the bank angle continues to decrease at a slower rate until the aircraft is close to the headwind point, after which the bank angle is held fairly constant at its minimum value.
(4) After passing point 3, the bank angle is held fairly constant for a short angular distance in travelling from point 3 to 4, after which the bank angle starts to increase at a faster rate as the aircraft approaches point 4. Again, the bank angle increase would be less than 3 degrees per 10 degrees of angular movement around the circle.
(5) After passing point 4, the bank angle will continue to increase at a slower rate until the bank angle is again at its maximum near point 1. This maximum bank angle will be held for a short angular distance past point 1.
After completing step (5), steps (1)-(5) would be repeated.

Up to this point in the discussion, we have not considered which variables the Pilot should be focusing on during the execution of this maneuver. Assuming the TAS is fixed, there are four variables that can be followed during the execution of the Turn Around a Point. These are:

(a) Radius of the turn
(b) Groundspeed
(c) Wind correction angle
(d) Bank angle

Since we are attempting to fly a constant radius around the point, the primary variable that we should be focusing on is the radius of the turn. The groundspeed is also a primary variable, in that changes in groundspeed are an indication of a required increase or decrease in bank angle. Although the wind correction angle is something that will tell us which direction the wind is blowing, it is not a variable that warrants a lot of the Pilot's attention. The argument is that if the aircraft is tracking directly over the circle, then the wind correction angle must be correct at each point around the circle. Since Figure 14 showed the importance of the "gradient bank angle" in performing a precision Turn Around a Point, we still require some information on how the Pilot is going to vary the rate of change in the bank angle as the aircraft proceeds around the circle. Since the groundspeed is a primary variable that the Pilot follows around the circle, we will now show that the rate of change of the groundspeed around the circle is a good indicator of the "gradient bank angle" parameter.

The rate of change of the groundspeed with respect to the angular position around the circle can be obtained by differentiating the groundspeed given in (36). The resulting expression is given by

$$\frac{d\bar{V}_G}{d\theta} = -\frac{\bar{V}_W}{Cos(\sigma)} Sin(\theta + \sigma) \qquad (53)$$

We can consider this expression to be the "gradient groundspeed" parameter. We can compare the "gradient bank angle" with the "gradient groundspeed", which is shown in Figure 15 for the three wind speed ratios of interest. The comparison is in relatively good agreement, with very close agreement on the headwind side of the circle and some deviation on the tailwind side of the circle. The deviation on the tailwind side of the circle is significantly smaller at the lower wind speed ratio. As the wind speed ratio increases, there is a slight angular shift between the peaks in the magnitude of both gradient parameters on the tailwind side of the circle. Therefore, it is clear that the "gradient groundspeed" parameter is representative of the "gradient bank angle", and can be used in place of the "gradient bank angle" parameter when flying the Turn Around a Point.

Thus, when flying the maneuver, the rate of change in the groundspeed can be used as an indication of how fast the bank angle will need to be increased or decreased. If the groundspeed is increasing at a faster rate, the bank angle will need to be increased at a faster rate. If the rate of increase in groundspeed is decreasing, a slower increase in bank angle will be required, and vice versa.

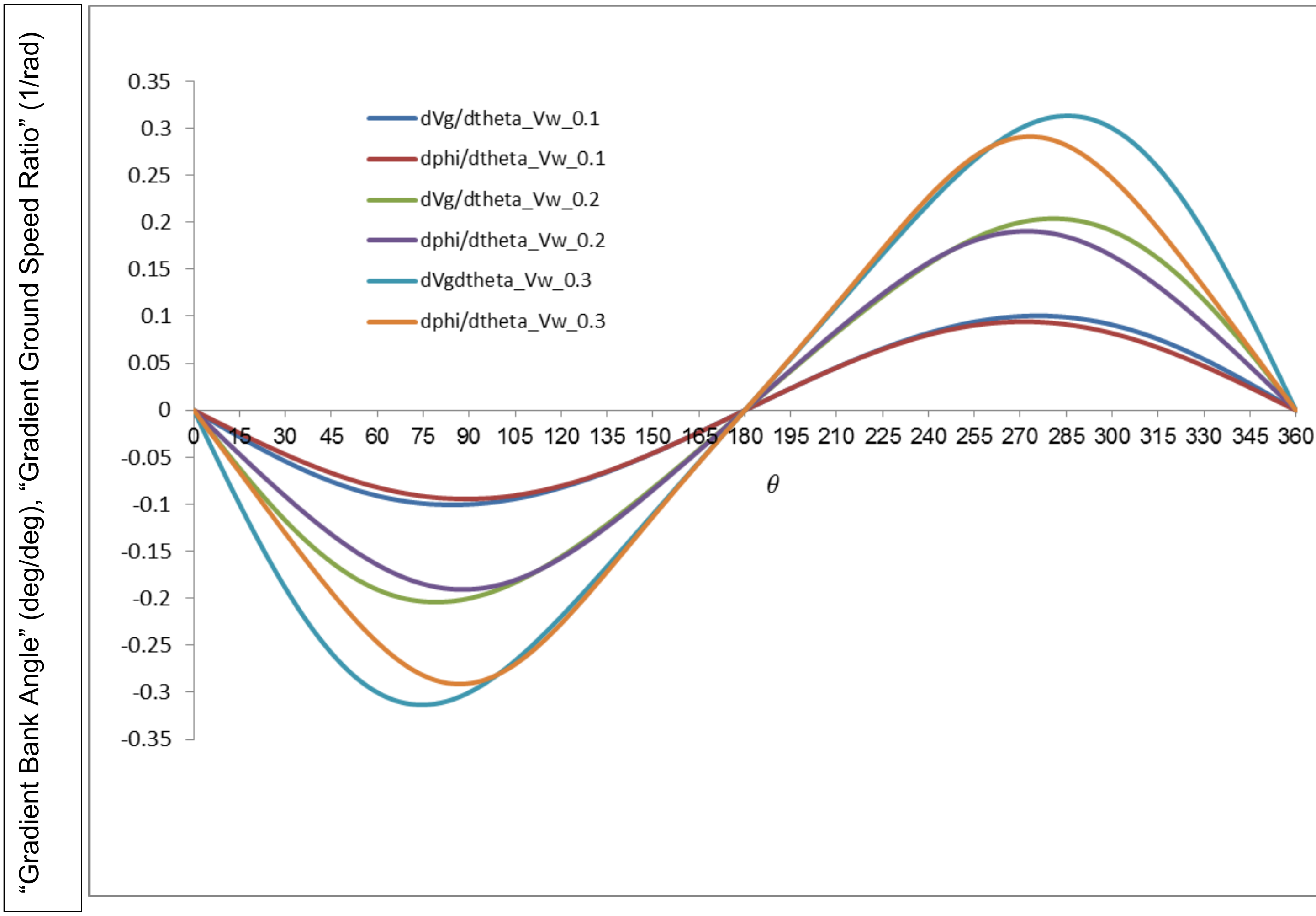

Figure 15: Comparison of the “Gradient Bank Angle” and “Gradient Groundspeed” Parameters Versus θ No-Wind Bank Angle 35 Degrees

7.0 Suggestions for Setting Up the Turn Around a Point

In Section 6 we discussed the fine points of executing a precision Turn Around a Point. We now provide some important details of how to select the radius of the turn which will keep the maximum bank angle at 45 degrees. The procedure we use is described below:

(a) Select a TAS that will be used to fly the maneuver. One can use the computer on the airspeed indicator to show TAS, by inputting the OAT and the pressure altitude.

(b) For the given wind velocity, compute the wind speed ratio

(c) Equation (47) can be used to determine the no-wind bank angle
(d) Equation (26) can then be used to determine the radius of the turn corresponding to the TAS and no-wind bank angle.
(e) Fly an altitude equal to the turn radius and project a 45 degree angle from the horizontal to the ground, such that your 45 degree line of sight intersects the center point of the circle. Once the radius is correct, the altitude can be changed to be between 600-1000AGL.

It is important that one fly at the highest altitude possible from the standpoint of safety, but low enough that would allow the Pilot to observe changes in the groundspeed of the aircraft.

Other suggestions for the Flight Instructor when training Student Pilots in Turns Around a Point are

(a) Practice the maneuver with a very light wind (i.e., no-wind) using a bank angle of both 35 and 45 degrees. In this case the bank angle will be nearly constant.
(b) Practice the maneuver with a wind speed of up to 25 knots using a no-wind bank angle of about 35 degrees, which will keep the maximum bank angle near 45 degrees.
(c) Use a different TAS with the same wind speed to demonstrate to the Student Pilot that the wind speed ratio is the key parameter which will control the both maximum bank angle and the “gradient bank angle” around the circle. In this case, the radius of the turn will need to decrease, since we need to keep the no-wind bank angle the same.

With the knowledge gained from this White Paper, both the Flight Instructor and the Student Pilot should be able to enter the maneuver at any point on the circle. However, if not entering on either the tailwind or the headwind points, the Pilot will need to have the proper wind correction angle on the entry. The wind correction angle on the upwind side or the downwind side was previously shown to be equal to approximately sixty times the wind speed ratio, i.e. equation (19), with the corresponding bank angle being nearly equal to the no-wind bank angle. This should be a very close first approximation to getting the aircraft to track the circle correctly. However, since the wind correction angle is zero on both the headwind and tailwind, it is easier to enter on either the headwind or the tailwind, Although the “Flight Training Handbook” recommends entering on the tailwind, since the aircraft will be at its maximum bank angle, entering on the upwind does have the advantage that the initial bank angle will be shallower and thus, the entry would be much smoother. Thereafter, the bank angle would be increased by no more than 3 degrees per every 10 degrees of angular movement around the circle.

8.0 Conclusions

In this White Paper we have derived the exact solution to the Turn Around a Point maneuver. The solution provides (1) The ratio of the groundspeed to the TAS, (2) the wind correction angle, (3) the bank angle, and (4) the rate of change in bank angle as a function of the angular position measured from the point when the aircraft is on the tailwind. The solution is a one-parameter family of curves, where the parameter is the wind speed ratio, i.e. the ratio of the wind speed to the TAS. The required Pilot input is the bank angle and rate of change in bank angle as a function of the angular position from the tailwind location. Based on these results, recommendations on how to fly the maneuver with greater precision are discussed. One key result is that the required rate of turn to stay on the circle can be closely approximated by the square of the groundspeed. Thus, using changes in groundspeed as the main cue for changes in bank angle is appropriate in maintaining the track around the circle. Universal curves for wind speed ratios of 0.1, 0.2 and 0.3 are provided, which are very useful for Flight Instructors in training Student Pilots on this maneuver.

The required turn radius for the Turn Around a Point maneuver, which keeps the maximum bank angle at 45 degrees on the downwind, has also been derived. It has been shown that the ratio of the required radius of the turn to the no-wind turn radius is just given by $(1+\bar{V}_w)^2$. This information is extremely useful to the Flight Instructor in setting up the maneuver.

Finally, it is important for all Flight Instructors and Pilots to understand that for wind speeds less than 30 knots, the rate of change of bank angle with angular distance around the circle is less than 3 degrees of bank angle change for every 10 degrees of angular movement around the circle. Therefore prior to performing the Turn Around a Point, the Student Pilot should have become proficient in turning flight during which the bank angle is "gradually" increased and decreased. Again, the term "gradually" has now been quantified.