A Short Note on the Turn Dynamics of a Turn around a Point Maneuver

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Summary

In a previous White Paper, Ref. 1, we derived the turn dynamics for the Turn around a Point maneuver. Here, we employed the concept of the Wind Triangle and some geometric arguments. In a companion White Paper, Ref. 2, we derived the turn dynamics for the On-Pylon Turn maneuver using the method of Classical Dynamics. In this Note, we use the results in Ref. 2 for the general expression for the Centripetal acceleration to derive the turn dynamics for the Turn around a Point maneuver. We show that both methods give the identical turn dynamics for a Turn around a Point maneuver, however, the Classical Dynamics approach is more consistent with the way we teach basic aerodynamics for the Private Pilot Program. Expressions annotated in red are important takeaways that the reader should comprehend in order to attain a better understanding of the Turn around a Point maneuver.

1.0 Classical Dynamics Approach

In a non-uniform rotating radial-transverse coordinate system (Ref. 2), the equation for the vector acceleration $\vec{a}$ is given by

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{i}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\mathbf{i}}_\theta \qquad (1)$$

Where r is the radius from the center of the turn, $\dot{r}$ and $\ddot{r}$ are the first and second derivatives of the radius with respect to time, and $\dot{\theta}$ is the aircraft non-uniform rotation rate. Equation (1) provides the resultant acceleration in both the radial ($\hat{i}_r$), and transverse ($\hat{i}_\theta$) directions. The radial component is called the Centripetal acceleration, and the transverse component is the acceleration in the transverse direction. However, since the coordinate system is rotating with the aircraft non-uniform rotation rate $\dot{\theta}$, the transverse acceleration relative to the rotating coordinate system can be shown to be identically zero in the case of the aircraft flying at constant TAS. Note that in the case of a Turn around a Point, the radius is constant, i.e. the rate of change of r with respect to time is identically zero, and thus, both $\dot{r}$ and $\ddot{r}$ are zero. This simplifies the Centripetal acceleration term, which now becomes

$$a_r = -r\dot{\theta}^2 \quad (2)$$

Where

$$\dot{\theta} = \frac{V_\theta}{r} \quad (3)$$

Here $V_\theta = V_G$ is the transverse groundspeed around the circle. Note the negative sign in eq. (2) indicates that the radial acceleration is directed toward the center of the circle. Thus, the final expression for the Centripetal acceleration is given by

$$a_r = -\frac{V_G{}^2}{r} \quad (4)$$

In the Turn around a Point maneuver, the aircraft develops a wind correction angle (WCA) in order to stay on the circle, and thus, the lateral axis of the aircraft is crabbed into the wind. If the wind correction angle is denoted by σ, then the transverse groundspeed, V_G is given by

$$V_G = V_{TAS}\,\mathrm{Cos}\,\sigma + V_W\,\mathrm{Cos}\,\theta \quad (5)$$

Where $\theta = 0$ degrees corresponds to the aircraft on the downwind, and $\theta = 180$ degrees corresponds to the aircraft on the upwind. Note that eq. (5) requires the WCA σ. Since the Turn around a Point is flown with a constant radius, the WCA can be determined by balancing the radial component of the wind with the radial component of the TAS, i.e. V_{TAS} multiplied by $\mathrm{Sin}\,\sigma$. This relationship is shown in eq. (6)

$$V_{TAS}\,\mathrm{Sin}\,\sigma = V_W\,\mathrm{Sin}\,\theta \quad (6)$$

Thus, the WCA is given by

$$WCA = \sigma = \mathrm{Sin}^{-1}(\bar{V}_W\,\mathrm{Sin}\,\theta) \quad (7)$$

Where $\bar{V}_W$ is defined as the windspeed ratio and is given by $\frac{V_W}{V_{TAS}}$. If the windspeed ratio $\bar{V}_W \le 0.5$, the wind correction angle σ in degrees, can by closely approximated by

$$\sigma = 60\bar{V}_W\,\mathrm{Sin}\,\theta \quad (8)$$

Note that eqs. (5) and (6) are identical to the solution of the Wind Triangle Problem given by eq. (20) in Ref.1, where the symbol α has been replaced by θ.

In order to derive the turn dynamics for the Turn around a Point maneuver, we equate the Centrifugal force to the horizontal component of the lift that is directed toward the center of the circle. The Centrifugal force (C.F.) is just the mass of the aircraft times the magnitude of the Centripetal acceleration, i.e.

$$C.F. = (\frac{W}{g})\frac{V_G^{\ 2}}{r} \qquad (9)$$

Where the mass of the aircraft is just the weight of the aircraft divided by the acceleration of gravity. Figure 1 shows the horizontal component of lift is given by $L\,\mathrm{Sin}\,\phi$, where ϕ is the bank angle. However, since the aircraft is crabbed into the wind the horizontal component of the lift does not point toward the center of the circle. The horizontal component of the lift that is directed toward the center of the circle is just $L\,\mathrm{Sin}\,\phi\,\mathrm{Cos}\,\sigma$ which is also depicted in Figure 1.

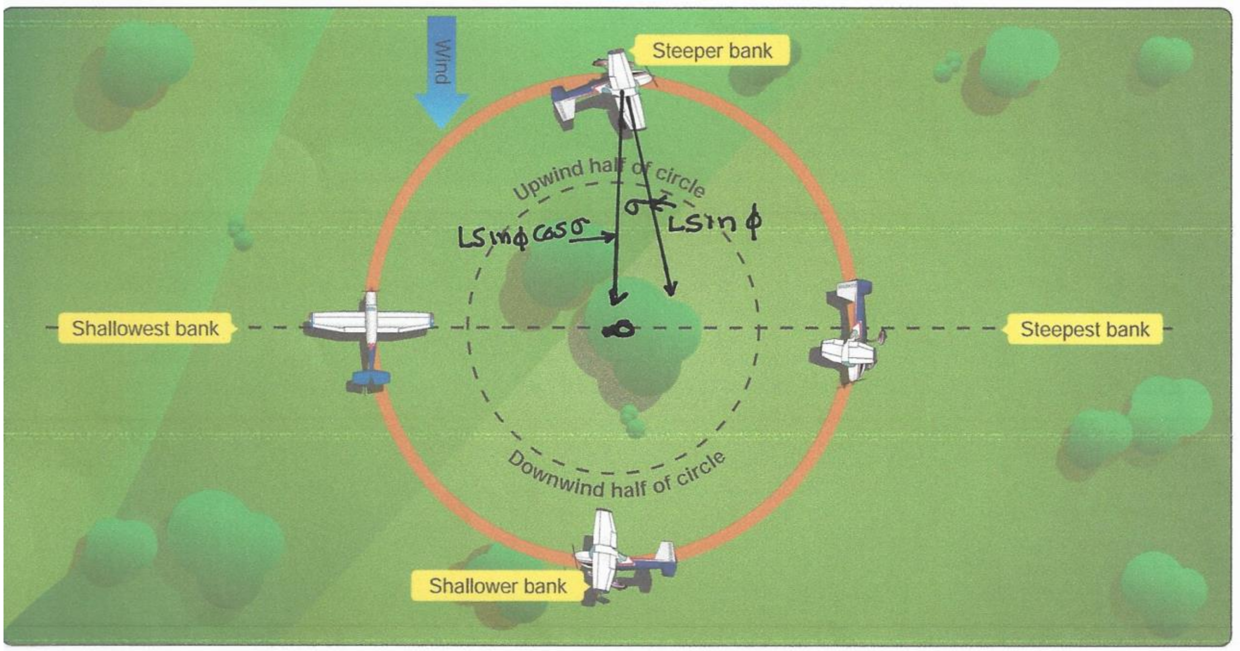

Figure 1: Horizontal Component of Lift Directed Toward the Center of the Circle

Since the Turn around a Point maneuver is flown at constant altitude, a balance of the vertical forces during the turn gives the following relationship between the lift and the weight

$$L\,\mathrm{Cos}\,\phi = W \qquad (10)$$

Equation (10) gives the ratio of the lift to the weight during the turn, i.e.

$$\frac{L}{W} = \frac{1}{\mathrm{Cos}\,\phi} = n \qquad (11)$$

Where n is the load factor on the aircraft during the turn. If we now equate the Centrifugal force to the horizontal component of the lift that is directed toward the center of the circle, we obtain the following equation for the required bank angle

$$\text{Tan}\,\phi = \frac{V_G^{\ 2}}{gr\,\text{Cos}\,\sigma} \qquad (12)$$

Normalizing the groundspeed by the V_{TAS}, i.e. $\bar{V}_G = \frac{V_G}{V_{TAS}}$, results in

$$\text{Tan}\,\phi = \left(\frac{V_{TAS}^{\ 2}}{gr}\right)\frac{\bar{V}_G^{\ 2}}{\text{Cos}\,\sigma} \qquad (13)$$

Since the first term in parenthesis on the right hand side of eq.(13) is the required bank angle under no wind conditions (i.e. ϕ_{NW}), we can write the above equation as

$$\frac{\text{Tan}\,\phi}{\text{Tan}\,\phi_{NW}} = \frac{\bar{V}_G^{\ 2}}{\text{Cos}\,\sigma} \qquad (14)$$

Where the normalized groundspeed $\bar{V}_G$ is given by

$$\bar{V}_G = \text{Cos}\,\sigma + \bar{V}_W\,\text{Cos}\,\theta \qquad (15)$$

Here again, the WCA σ is given by

$$\sigma = \text{Sin}^{-1}(\bar{V}_W\,\text{Sin}\,\theta) \qquad (16)$$

Note that on the downwind ($\theta = 0$) and upwind ($\theta = 180$), $\text{Sin}\,\theta = 0$, and the WCA is identically zero. It is clear that the maximum value of the WCA occurs on the crosswind (i.e. $\theta = 90$ or 270 degrees), where the magnitude of the WCA is given by $\sigma = \text{Sin}^{-1}(\bar{V}_W)$.

An important takeaway from eq. (13) concerns constraining the aircraft's maximum bank angle on the downwind. If one is interested in constraining the bank angle on the downwind to a particular maximum value, the value of the radius of the circle can be determined for a given value of V_{TAS}. For example, if the maximum allowable bank angle on the downwind is to be 45 degrees, $\text{Tan}\,\phi = 1$, and thus, the required radius of the circle is given by

$$r = \frac{V_{TAS}^{\ 2}}{g}\frac{\bar{V}_G}{\text{Cos}\,\sigma} \qquad (17)$$

On the downwind, the WCA is identically zero, and $\bar{V}_G = 1 + \bar{V}_W$. Therefore the required radius of the circle is given by

$$r = \frac{V_{TAS}^{\ 2}}{g}(1 + \bar{V}_W)^2 \qquad (18)$$

Note that the no wind radius for a 45 degree banked Turn around a Point maneuver is given by

$$r_{NW} = \frac{V_{TAS}^{\ 2}}{g} \qquad (19)$$

Therefore, in order to constrain the maximum bank angle on the downwind to 45 degrees, the required radius of the turn in the presence of a wind is given by

$$r = r_{NW}(1+\bar{V}_W)^2 \qquad (20)$$

Thus, for a windspeed ratio of 0.2, in order to constrain the bank angle on the downwind to no more than 45 degrees, the required radius of the turn will be 1.44 times the no wind radius r_{NW}, The information discussed above will aid the Pilot/Instructor in setting up the maneuver under any wind condition.

Finally, since eq. (14) is identical to eq. (46) in Ref. 1, we have shown that both methods give the same equation for the turn dynamics of a Turn around a Point maneuver. However, the Classical Dynamics method is more consistent with the method of teaching basic aerodynamics in the Private Pilot Program. It is important that all Pilots and Instructors understand the actual complexity of the variation of the bank angle during the Turn around a Point maneuver, which allows the aircraft to maintain a constant radius circle around a selected point on the surface (see Ref. 1).

2.0 References

(1) Glatt, L., “A New and Novel Approach for Understanding and Flying a Precision Turn around a Point”, SAFE Member Resource Center, April 2014.

(2) Glatt, L., “A New and Novel Approach for Understanding and Flying a Precision On-Pylon Turn”, SAFE Member Resource Center, August 2017.